

## DEFLECTION OF CONCRETE FLOOR SYSTEMS FOR SERVICEABILITY<sup>1</sup>

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Deflection control is a central considerations in serviceability of floor systems. This Technical Note reviews the levels of acceptable deflections and the currently available methods for their estimate.

### OVERVIEW

There are several reasons to control deflection.

- ❖ A concrete floor should have adequate stiffness to prevent changes in deflection that would damage attached partitions or other construction elements likely to be damaged by large deflections.
- ❖ The deflection of a floor should not be noticeable by occupants such as to convey a sense of inadequacy or safety concerns.
- ❖ Since, in some instances, deflection is used as a measure leading of undesirable vibration in a floor, its value must be controlled.

### LIMITS FOR ACCEPTABLE DEFLECTION

#### **Aesthetics and Sense of Comfort**

In considering aesthetics and sense of comfort for occupants, the most important criterion is the out-of-level condition of a floor, as opposed to its stiffness. Sensitive individuals, when walking over or viewing a floor in elevation, are claimed to perceive a floor's sag when the vertical out-of-level to span ratio is in excess of 1/250, and for cantilevers in excess of 1/125. The out-of-level condition of a floor system can be controlled through camber at the time of construction, upon estimating the long-term deflection.

#### **Deflection Limits to Mitigate Damage to Non-structural Construction**

It is important to note that ACI [ACI 318, 2008] does not impose a limit to deflection under selfweight. ACI's recommendations address the amount of deflection subsequent to the installation of non-structural elements likely to be damaged.

The following table lists the ACI's stipulation on deflections (TABLE 9.5(b)). In the application of ACI's recommended deflection limits, it is important to recognize that the given values are to be compared with "computed" values, not measured out-of-plane amounts.

TABLE 1 MAXIMUM PERMISSIBLE COMPUTED DEFLECTIONS

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Type of member	Deflection to be considered	Deflection limitation
Flat roofs not supporting or attached to nonstructural elements likely to be damaged by large deflection	Immediate deflection due to live load	L/180 *
Floors not supporting or attached to nonstructural elements likely to be damaged by large deflection	Immediate deflection due to live load	L/360
Roof or floor construction supporting or attached to nonstructural elements likely to be damaged by large deflection	That part of the total deflection occurring after attachment of nonstructural elements(sum of the long-time deflection due to all sustained loads and the immediate deflection due to any additional live load)****	L/480 **
Roof or floor construction supporting or attached to nonstructural elements not likely to be damaged by large deflection		L/240 ***

**Notes:**

- \* Limit not intended to safeguard against ponding. Ponding should be checked by suitable calculations of deflection, including added deflections due to ponding of water, and considering long-term effects of all sustained loads, camber, construction tolerances, and reliability of provisions for drainage.
- \*\* Limit may be exceeded if adequate measures are taken to prevent damage to supported or attached elements.
- \*\*\* But not greater than tolerance provided for nonstructural elements. Limit may be exceeded if camber is provided so that total deflection minus camber does not exceed limit.
- \*\*\*\* Long-time deflection shall be determined using established procedures, but may be reduced by amount of deflection calculated to occur before attachment of nonstructural elements. This amount shall be determined on basis of accepted engineering data relating to time-deflection characteristics of members similar to those being considered.

**DEFLECTION CONTROL THROUGH LIMITATIONS ON SPAN TO DEPTH RATIOS**

For common residential and commercial buildings, designers can forego deflection calculation, if the stiffness of the member selected is large enough. Deflection calculation requirements are governed through recommended span-to-depth ratios for different types of floor members. ACI 318 has the recommendations given in Tables 2 and 3.

**One-Way Conventionally Reinforced Slabs and Beams**

**TABLE 2 MINIMUM THICKNESS OF CONVENTIONALLY REINFORCED BEAMS OR ONE-WAY SLABS**

<b>Member</b>	<b>Simply supported</b>	<b>One end continuous</b>	<b>Both ends continuous</b>	<b>Cantilever</b>
Solid one-way slabs	L/20	L/24	L/28	L/10
Beams or ribbed one-way slabs	L/16	L/18.5	L/21	L/8

Notes:

L =span length

Values given shall be used directly for members with normal weight concrete and Grade 60 ksi (400 MPa) reinforcement. For other conditions, the values shall be modified as follows:

- a) For lightweight concrete having equilibrium density,  $w_c$ , in the range of 90 to 115 lb/ft<sup>3</sup> (1440-1840 kg/m<sup>3</sup>), the values shall be multiplied by (1.65-0.005  $w_c$ ) but not less than 1.09 [in SI units, (1.65-0.003 $\gamma_c$ ) but not less than 1, where  $\gamma_c$  is the density in kg/m<sup>3</sup>].
- b) For  $f_y$  other than 60,000 psi(400 MPa), the values shall be multiplied by (0.4+ $f_y/100,000$ ) [in SI units (0.4+ $f_y/670$ )].

**Two-Way Conventionally Reinforced Slabs and Beams**

**TABLE 3 MINIMUM THICKNESS OF SLABS WITHOUT INTERIOR BEAMS\***

<b><math>f_y</math> psi**</b>	<b>Without drop panels***</b>			<b>With drop panels***</b>		
	<b>Exterior panels</b>		<b>Interior panels</b>	<b>Exterior panels</b>		<b>Interior panels</b>
	<b>Without edge beams</b>	<b>With edge beams****</b>		<b>Without edge beams</b>	<b>With edge beams****</b>	
40,000	Ln/33	Ln/36	Ln/36	Ln/36	Ln/40	Ln/40
60,000	Ln/30	Ln/33	Ln/33	Ln/33	Ln/36	Ln/36
75,000	Ln/28	Ln/31	Ln/31	Ln/31	Ln/34	Ln/34

Notes:

\* For two-way construction,  $L_n$  is the length of clear span in the long direction, measured face-to-face of supports in slabs without beams and face-to-face of beams or other supports in other cases.

\*\* For  $f_y$  between the values given in the table, minimum thickness shall be determined by linear interpolation.

\*\*\* Drop panels are defined as extension of slab thickening into span not less than span/6, and extension of thickening below slab not less than slab thickness/4.

\*\*\*\* Slabs with beams between columns along exterior edges. The ratio of edge beam stiffness to the stiffness of the edge beam’s design strip shall not be less than 0.44

**Post-Tensioned Members**

For post-tensioned beams and slabs, the recommended values by the Post-Tensioning Institute [PTI, 1990] are as follows:

TABLE 4 RECOMMENDED SPAN TO DEPTH RATIOS FOR POST-TENSIONED MEMBERS

	Continuous Spans		Simple Spans	
	Roof	Floor	Roof	Floor
<b>One-way solid slabs</b>	50	45	45	40
<b>Two-way solid slabs (supported on columns only)</b>	45-48	40-45		
<b>Two-way waffle slabs (1m pans)</b>	40	35	35	30
<b>Beams</b>	35	30	30	26
<b>One-way joists</b>	42	38	38	35

Note: The above ratios may be increased if calculations verify that deflection, camber, and vibrations are not objectionable.

**DEFLECTION CALCULATIONS**

Under otherwise unchanged conditions, the deformation of an exposed and loaded concrete member continues to increase. The increase is due to creep under applied load and shrinkage from loss of moisture. The engineering approach to estimating of deflection is to determine the instantaneous response of a structure under an applied load, and magnify the instantaneous displacement due to the time-dependent factors of creep and shrinkage. With time, the rate of change in displacement reduces. For building structure it is assumed that five years is sufficient time for the deflections to have reached their final values. While it is practical to calculate the time-dependent deflection for any time interval, the common practice is to estimate the total value at five years and use this value in the design.

**Instantaneous Deflection**

Instantaneous deflection is generally calculated using concrete’s modulus of elasticity at 28 days, gross-cross sectional area and linear elastic theory. The calculated deflection may require adjustment, if the member is likely to crack, when subjected to the design load. Cracking reduces the stiffness of a member and results in increased deflection. The options for calculating instantaneous deflection with due allowance to cracking are:

- ❖ Closed form formulas or tables, available primarily for uncracked sections;
- ❖ Use of equivalent moment of inertia ( $I_e$ ) and simplified averaging (ACI-318’s simplified procedure);
- ❖ Use of equivalent moment of inertia ( $I_e$ ) combined with numerical integration; and
- ❖ Use of Finite Element floor programs that allow for cracking.

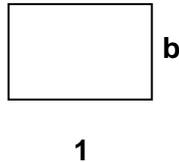
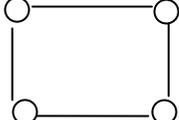
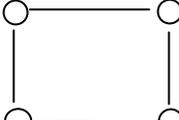
Each of the above procedures is briefly discussed in the following section.

**Closed Form Formulas**

Closed form formulas are readily available for beams and one-way slabs. The variables that describe the geometry of a two-way panel within a floor system, however, are so extensive that it becomes impractical to compile a meaningful set of tables or relationships without extensive approximation. For non-cracked sections, compilations such as the one listed in Table 5 are readily available in the literature [Bares, 1971].

In the application of data, such as those given in Table 5, the design engineer must use judgment regarding the degree of fixity of the support.

TABLE 5 DEFLECTION COEFFICIENTS k

$\gamma$				
1	0.0457	0.0143	0.0653	0.0491
1.1	0.0373	0.0116	0.0548	0.0446
1.2	0.0306	0.0094	0.0481	0.0422
1.3	0.0251	0.0075	0.0436	0.0403
1.4	0.0206	0.0061	0.0403	0.0387
1.5	0.0171	0.0049	0.0379	0.0369
2.0	0.0071	0.0018	0.0328	0.0326

Notes:

Poisson's ratio conservatively assumed 0.25

$\gamma = a/b$  (aspect ratio)

Boundary conditions

- 1 = rigid supports; rotationally free;
- 2 = rigid supports; rotationally fixed;
- 3 = central panel from an array of identical panels supported on columns; deflection at center; and
- 4 = similar to case 3, but deflection at center of long span at support line

$$w = k (a^4 \cdot q / E \cdot h^3)$$

Where,

- w = deflection normal to slab;
- a = span along X-direction;
- E = Modulus of elasticity; and
- h = slab thickness.

**EXAMPLE 1**

Consider the floor system shown in Fig. EX-1. Estimate the deflection of the slab panel identified in part b of the figure under the following conditions. Other particulars of the floor system are noted in Appendix A.

Given:

Span length along X-X direction	= 30' (9.14 m)
Span length along Y-Y direction	= 26.25' (8.0 m)
Slab thickness	= 8 in (203 mm)
$E_c$ (modulus of elasticity)	= $4.287 \times 10^6$ psi (29,558 MPa)

Superimposed dead load	= 25 psf (1.2 kN/m <sup>2</sup> )
Live load	= 40 psf (1.9 kN/m <sup>2</sup> )

Required

Deflection of the panel at midspan for the following load combination

$$1*DL + 1*LL$$

$$\text{Aspect ratio } \gamma = 30/26.25 = 1.14$$

$$\text{Total service load } q = [(20+5+40) + 150*8/12]/144 = 1.146 \text{ lb/in}^2 (7.9*10^{-3} \text{ N/mm}^2)$$

**Using closed form formulas (Table 5)**

$$(a^4*q / E*h^3) = [(30*12)^4 * 1.146 / (4.287*10^6 * 8^3)] = 8.77 \text{ in (222.76 mm)}$$

For mid-panel deflection, consider case 3 from Table 5

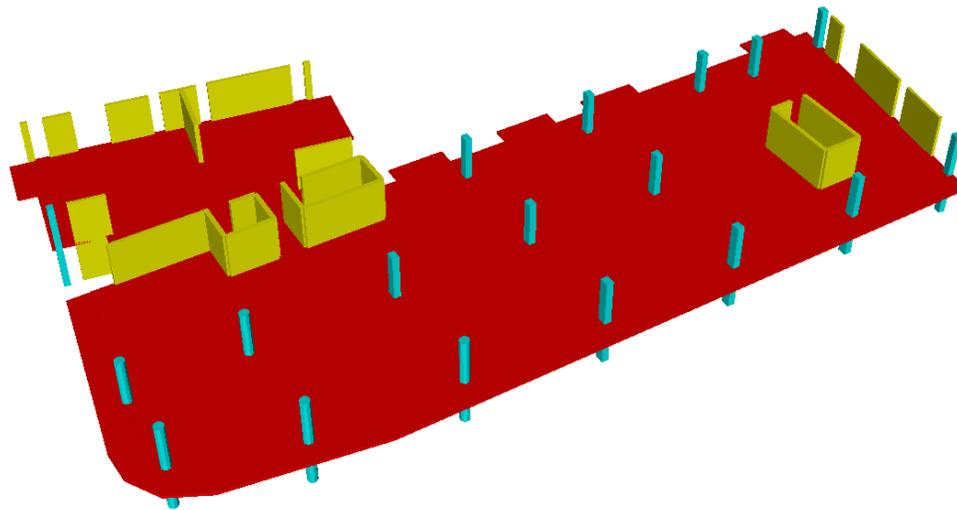
$$k = 0.0548$$

$$\text{Deflection, } \Delta = k (a^4*q / E*h^3) = 0.0548*8.77 = 0.48 \text{ in (12.20 mm)}$$

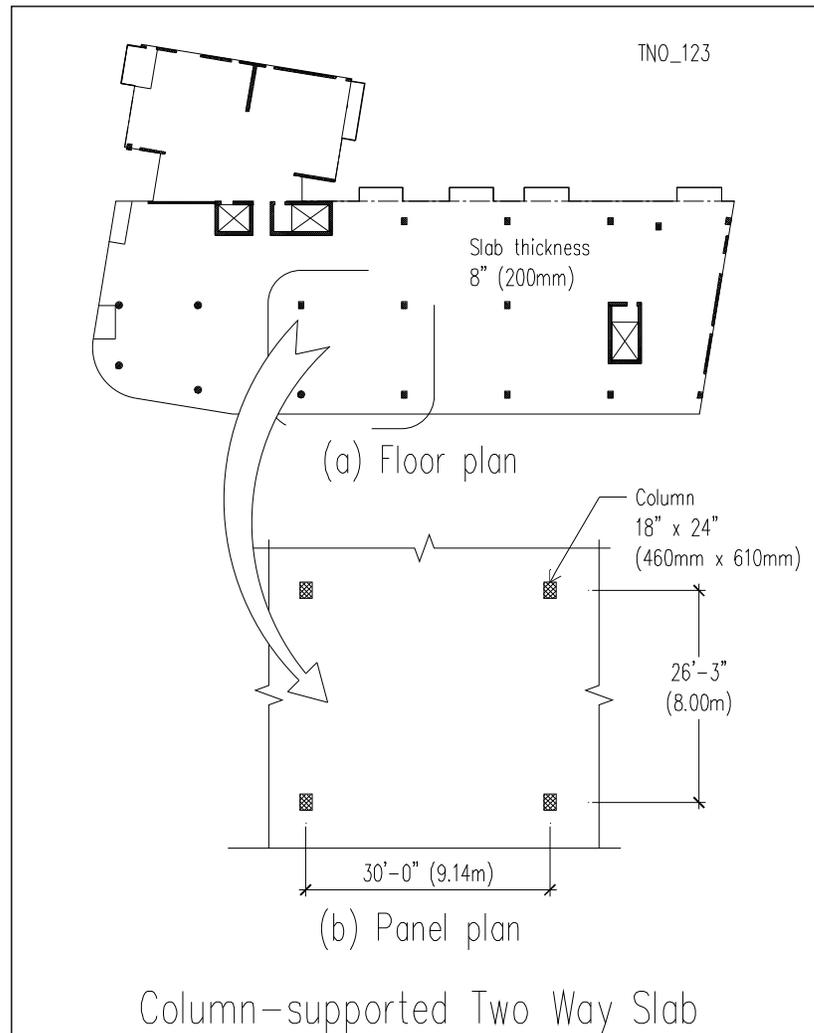
For deflection at midpoint of column lines in X-direction, from Table 5

$$k = 0.0446$$

$$\text{Deflection, } \Delta = k (a^4*q / E*h^3) = 0.0446*8.77 = 0.39 \text{ in (9.91 mm)}$$



(a) 3D View of typical floor system



(b) Illustration of design panel

FIGURE EX-1 TYPICAL FLOOR HIGHLIGHTING THE SPAN UNDER CONSIDERATION

### Using equivalent moment of inertia ( $I_e$ ) and (ACI-318's simplified procedure)

In this method allowance is made for crack formation in the slab. The reduction in flexural stiffness due to cracking is accounted for by substituting the otherwise gross moment of inertia " $I_g$ " used in the calculation with a reduced effective moment of inertia " $I_e$ ." The equivalent moment of inertia  $I_e$  can be applied to the entire span through the "simplified procedure of ACI-318), or applied locally along the entire length of a member for a detailed procedure.

The calculation of the effective moment of inertia  $I_e$  will be described next.

Using ACI-318

$$I_e = (M_{cr} / M_a)^3 * I_g + [1 - (M_{cr} / M_a)^3] * I_{cr} \leq I_g \quad (1)$$

Where,

$I_g$	=	Gross moment of inertia;
$I_{cr}$	=	Moment of inertia of cracked section;
$I_e$	=	Effective moment of inertia;
$M_a$	=	Maximum moment in member at stage deflection is computed; and,
$M_{cr}$	=	Cracking moment.

The applied moment,  $M_a$ , is calculated using elastic theory and the gross moment of inertia ( $I_g$ ) for the uncracked section. The change in distribution of moment in indeterminate structures resulting from cracking in concrete is generally small, and is already accounted for in the empirical formula (1) for equivalent moment of inertia. The cracking moment is given by:

$$M_{cr} = f_r I_g / y_t \quad (2)$$

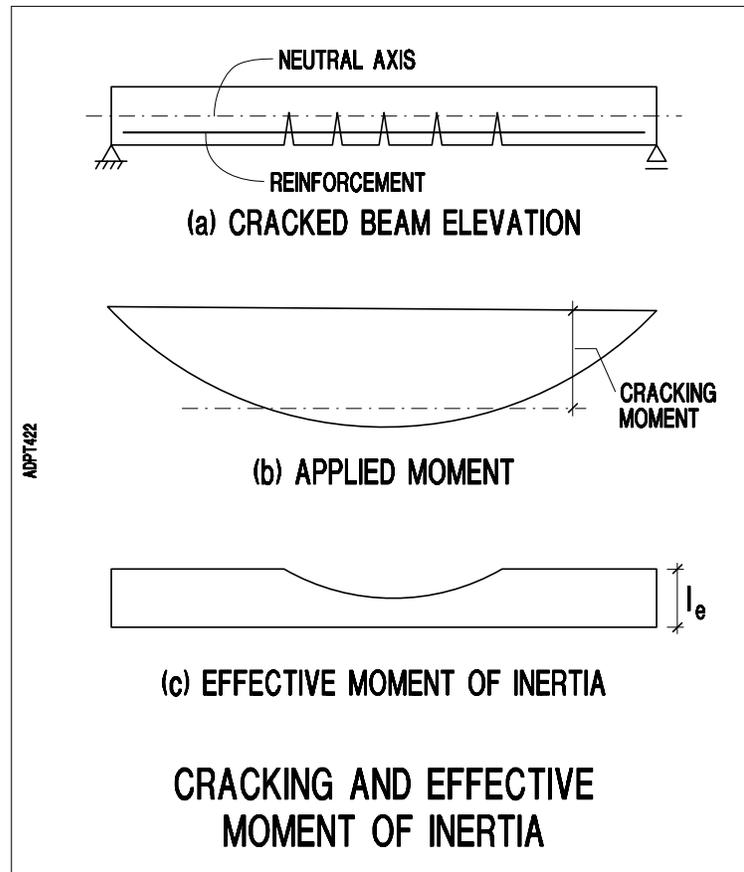
Where,

$f_r$	=	Modulus of rupture, flexural stress causing cracking. It is given by:
$f_r$	=	$7.5 f'_c{}^{1/2}$ <span style="float: right;">(3)</span>
$y_t$	=	distance of section centroid to farthest tension fiber

For all-lightweight concrete,  $f_r$  is modified as follows:

$$f_r = 0.75 * 7.5 f'_c{}^{1/2} \quad (4)$$

Figure 2 illustrates the equivalent moment of inertia  $I_e$  for a simply supported concrete slab that is partially cracked.



**FIGURE 2 ILLUSTRATION OF EFFECTIVE MOMENT OF INERTIAL  
IN A PARTIALLY CRACKED SLAB**

The value of cracking moment of inertial  $I_{cr}$  and the geometry of the section depends on the location and amount of reinforcement. For rectangular sections with single reinforcement (Fig. 2 ) the value is given by:

$$I_{cr} = (bk^3d^3)/3 + nA_s(d-kd)^2 \tag{5}$$

Where,

$$kd = [(2dB+1)^{1/2} - 1]/B \tag{6}$$

$d$  = distance from compression fiber to center of tension reinforcement

$$B = b/(nA_s)$$

$$n = E_s/E_c$$

$E_s$  = modulus of elasticity of steel

$E_c$  = modulus of elasticity of concrete

For more details and treatment of other cross-sections refer to ADAPT Technical Note TN293.

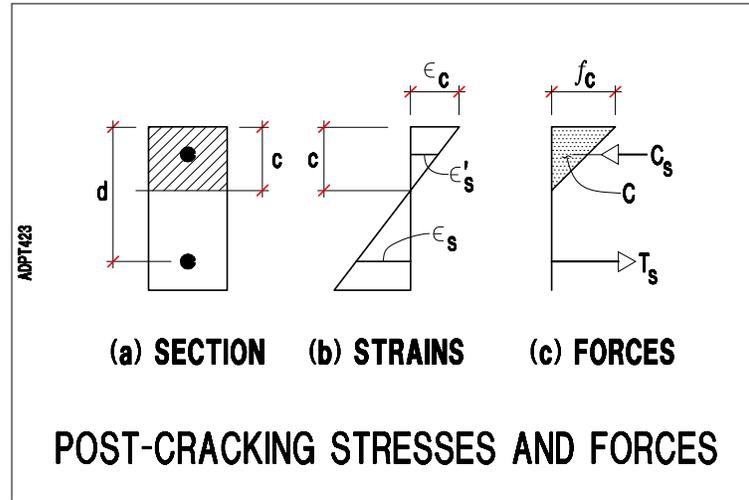


FIGURE 3

In the simplified method an average value of  $l_e$  is used for the entire span. For spans, the average value is calculated

$$l_{e, av} = 0.5 [ (l_{e, \text{left support}} + l_{e, \text{right support}}) / 2 + l_{e, \text{midspan}} ] \quad (7)$$

**EXAMPLE 2**

Consider the floor system shown in Fig. EX-1. Estimate the deflection of the slab panel identified under the same loading and conditions expressed in Example 1, using the simplified option of ACI-318 for equivalent moment of inertia  $l_e$

Given:

- Span length along X-X direction = 30' (9.14 m)
- Span length along Y-Y direction = 26.25' (8.0 m)
- Slab thickness = 8 in (203 mm)
- $E_c$  (modulus of elasticity) =  $4.287 \cdot 10^6$  psi (29558 MPa)
- Other details of the slab are given in Example 1 and the Appendix A

Required

Determine the deflection at the center of the panel identified in Example 1 due to the sum of dead and live loads.

Calculate Cracking Moment  $M_{cr}$

- $I_g$  = 15,360 in<sup>4</sup> (6.40e+10 mm<sup>4</sup>)
- $y_t$  = 4 " ( 101.60 mm)
- $f_r$  =  $7.5\sqrt{f_c} = 7.5\sqrt{5000} = 530.33$  psi (3.66 MPa)

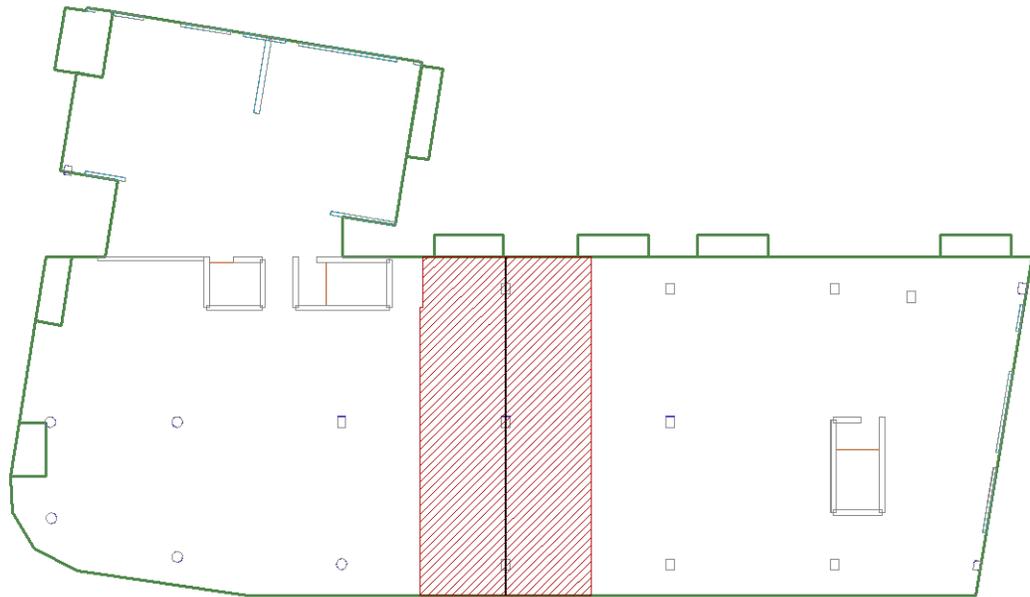
$$M_{cr} = f_r l_g / y_t = 530.33 * 15,360 / (4 * 12000) = 169.70 \text{ k-ft (230 kNm)}$$

To determine the deflection at center, the applied moment ( $M_a$ ) for the “design strip” associated with the panel in question must be determined. Refer to Fig. EX2 – 1a

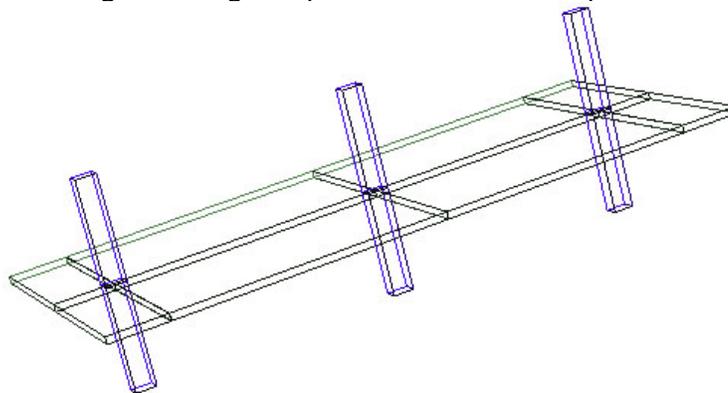
$$I_e = (M_{cr} / M_a)^3 * I_g + [1 - (M_{cr} / M_a)^3] * I_{cr} \leq I_g \tag{1}$$

The design strip associated with the panel under consideration is shown in Fig. EX-2a. It connects the line of columns and extends on each side to the midspan line of the adjacent panels.

The design strip extracted from the floor system is shown in its idealized form in Fig. EX-2b. Using a computer program, the applied moment  $M_a$  in the idealized design strip is calculated.



(a) Plan of slab showing the design strip associated with the panel under consideration



(b) View of the design strip extracted from the floor system

**FIGURE EX2 -1 PLAN OF TYPICAL FLOOR HIGHLIGHTING THE DESIGN STRIP OF THE SPAN UNDER CONSIDERATION**



It is noteworthy that the strip method, as outlined herein, provides ??

Using the moment values given in Fig. EX2-2 for the right support of span 1 the  $I_e$  is given by:

$$M_a = 326.4 \text{ k-ft (442.53 kNm)}$$

$$I_g = 17,019 \text{ in}^4 (6.40\text{e}+10 \text{ mm}^4)$$

$$M_{cr} = f_r I_g / y_t = 530.33 * 17,019 / (4 * 12000) = 188.04 \text{ k-ft (254.94 kNm)}$$

$$I_e = (M_{cr} / M_a)^3 * I_g + [1 - (M_{cr} / M_a)^3] * I_{cr} \leq I_g$$

Where,

$$I_{cr} = (b k^3 d^3) / 3 + n A_s (d - k d)^2$$

$$k d = [(2 d B + 1)^{1/2} - 1] / B$$

$$d = 6.81 \text{ in (173 mm)}$$

$$B = b / (n A_s)$$

$$n = E_s / E_c = 30000 / 4287 = 7.0$$

$$A_s = 10.12 \text{ in}^2 (6529 \text{ mm}^2)$$

$$B = 360 / (7.0 * 10.12) = 5.08 \text{ /in (0.2/mm)}$$

$$k d = [(2 * 6.81 * 5.08 + 1)^{1/2} - 1] / 5.08$$

$$= 1.45 \text{ in (36.83 mm)}$$

$$I_{cr} = (360 * 1.45^3) / 3 + 7.0 * 10.12 * (6.81 - 1.45)^2$$

$$= 2401 \text{ in}^4 (9.99\text{e}+8)$$

$$I_e = (188.04 / 326.4)^3 * 17019 + [1 - (188.04 / 326.4)^3] * 2401$$

$$= 5196 \text{ in}^4 (2.16\text{e}+9) = 0.31 I_g$$

Using the same procedure, the value of  $I_e$  at other locations required by the code formula are calculated and listed below:

Left cantilever:

$$I_e \text{ at face of support} = I_g = 1.536\text{e}+4 \text{ in}^4 (6.39\text{e}+9 \text{ mm}^4)$$

First Span:

$$I_e \text{ at left support centerline} = 1.70\text{e}+4 \text{ in}^4 (7.08\text{e}+9 \text{ mm}^4)$$

$$I_e \text{ at midspan} = 1.44\text{e}+4 \text{ in}^4 (5.99\text{e}+9 \text{ mm}^4)$$

$$I_e \text{ at right support centerline} = 5.20\text{e}+3 \text{ in}^4 (2.16\text{e}+9 \text{ mm}^4)$$

Second Span:

$$I_e \text{ at left support centerline} = 5.63\text{e}+3 \text{ in}^4 (2.34\text{e}+9 \text{ mm}^4)$$

$$I_e \text{ at midspan} = 1.536\text{e}+4 \text{ in}^4 (6.39\text{e}+9 \text{ mm}^4)$$

$$I_e \text{ at right support centerline} = 1.702\text{e}+4 \text{ in}^4 (7.08\text{e}+9 \text{ mm}^4)$$

Right cantilever:

$$I_e \text{ at face of support} = I_g = 1.536e+4 \text{ in}^4 \text{ (6.39e+9 mm}^4\text{)}$$

Using the averaging procedure suggested by ACI-318, the  $I_e$  values to be used for deflection calculation are:

Left and right cantilevers  $I_e = I_g$

First span

$$\begin{aligned} \text{Average } I_e &= [(1.70 \times 10^4 + 5.20 \times 10^3)/2 + 1.44 \times 10^4]/2 \\ &= 12.755e+3 \text{ in}^4 \text{ (5.31e+9 mm}^4\text{)} \end{aligned}$$

Second span

$$\begin{aligned} \text{Average } I_e &= [(5.63 \times 10^3 + 1.702 \times 10^4)/2 + 1.536 \times 10^4]/2 \\ &= 13.343e+3 \text{ in}^4 \text{ (5.55e+9 mm}^4\text{)} \end{aligned}$$

In order to use the same frame program for the calculation of deflected shape, the calculated equivalent moments of inertia are used to determine an equivalent thickness ( $h_e$ ) for each of the spans. The equivalent thickness is given by

$$I_e = b \cdot h_e^3 / 12$$

Where,  $b$  is the width of the tributary of the design strip.

Left cantilever:	$h_e = 8 \text{ in (203 mm)}$
First span	$h_e = 7.52 \text{ in (191 mm)}$
Second span	$h_e = 7.63 \text{ in (193.8 mm)}$
Right cantilever	$h_e = 8 \text{ in (203 mm)}$

Using the same computer program, material values, boundary conditions and loads, but with the reduced slab thickness of modified moment of inertia a new solution is obtained.

The maximum value of deflection for span 1 is 0.264 in (6.71 mm), compared to 0.231 in (5.87 mm), without allowing for reduction of stiffness due to cracking. Note that the above deflections do not account for the flexure of the slab in the orthogonal direction, as indicated in Example 2. For engineering design, where panels are fairly square, the calculated values are commonly multiplied by 2 to represent the deflection at the middle of panel. Hence, mid-panel deflections would be 0.528 in. (13.42 mm) and 0.462 in. (11.74 mm).

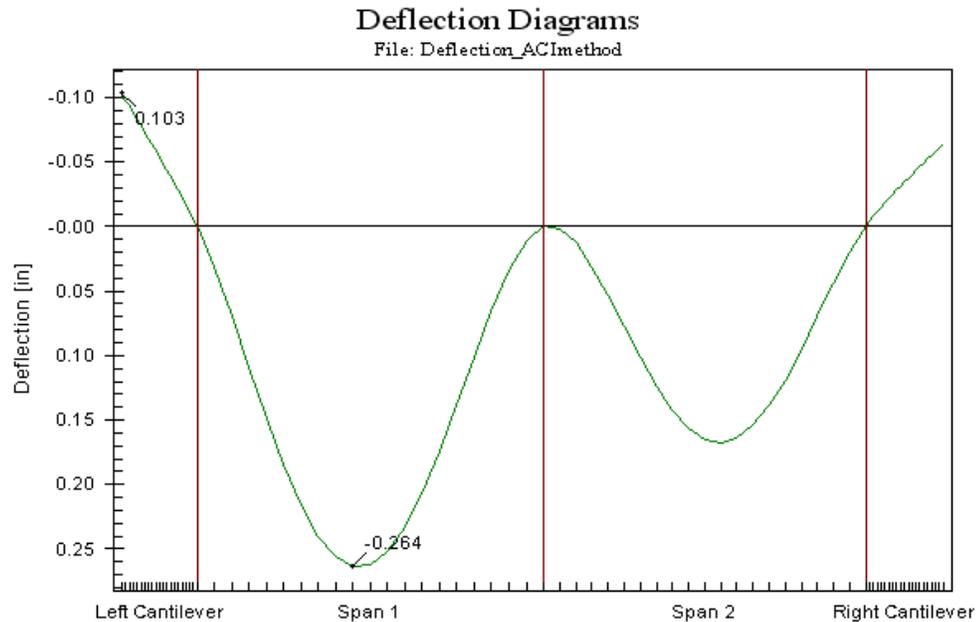


FIGURE EX2-3 DEFLECTED SHAPE WITH ALLOWANCE FOR CRACKING,  
USING SIMPLIFIED METHOD.

### Using Equivalent Moment of Inertia ( $I_e$ ) Combined with Numerical Integration

The next step in increased accuracy of deflection calculation is (i) the use of equivalent moment of inertia  $I_e$ , (ii) the strip method as outlined in the preceding example, and (iii) numerical integration. In this scheme each span will be subdivided in a number of segments, typically 10 to 20 divisions. The equivalent moment of inertia for each division will be calculated separately, and a solution obtained with recognition of a variable moment of inertia along the length of each span. This procedure along with a detailed numerical example is described in ADAPT TN294

Figure 3 is an example showing the variation of moment along the first span of a two-span member, subdivision of the span into smaller segments, and the equivalent moment of inertia for each segment due to cracking.

Using the computer program ADAPT-RC, the above procedure is employed to determine the deflection of the design strip shown in Fig. 4, with due consideration for cracking. The calculated deflection by the program is 0.235 in (5.97 mm), compared to 0.264 in (6.71 mm) where the simplified averaging of effective moment of inertia was used in calculation of cracked deflection.

Using this method, the total deflection is estimated as:

$$\text{Total deflection} = 2 * 0.235 = 0.470 \text{ in. (11.94 mm)}$$

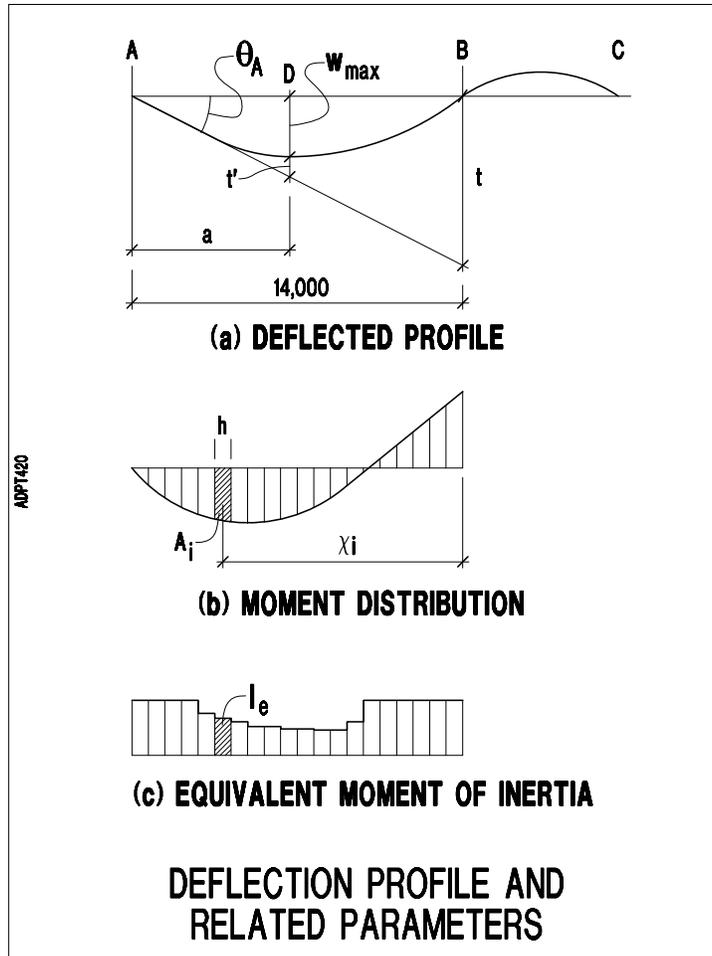


FIGURE 3 VARIABLE MOMENT INERTIA ALONG A MEMBER DUE TO CRACKING

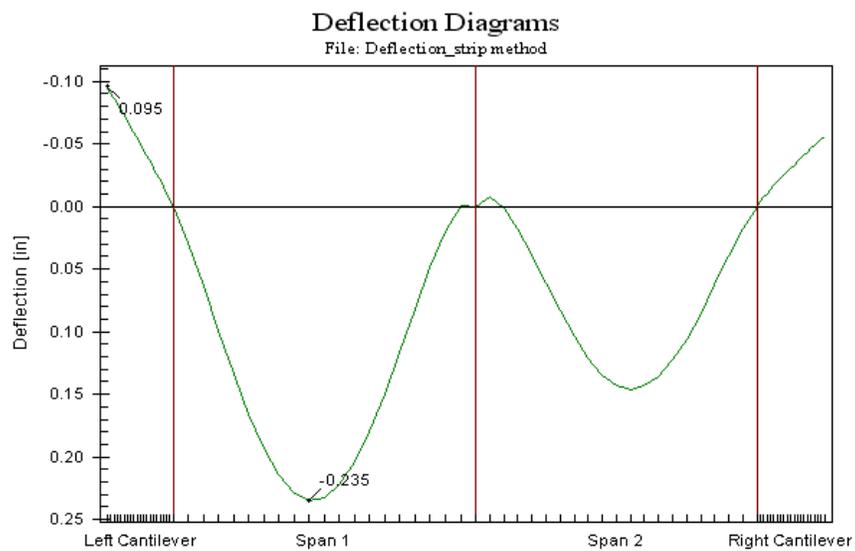


FIGURE 4 DEFLECTED PROFILE OF THE DESIGN STRIP WITH ALLOWANCE FOR CRACKING USING NUMERICAL INTEGRATION

### Using Finite Element Method With No Allowance for Cracking

Using finite element method (FEM), the salient features of the geometry and loading that are idealized in other previously explained options can be faithfully modeled. This leads to a more valid estimate of slab deflection. Figure 5 shows the Discretization of the floor system used in the previous examples into finite element cells.

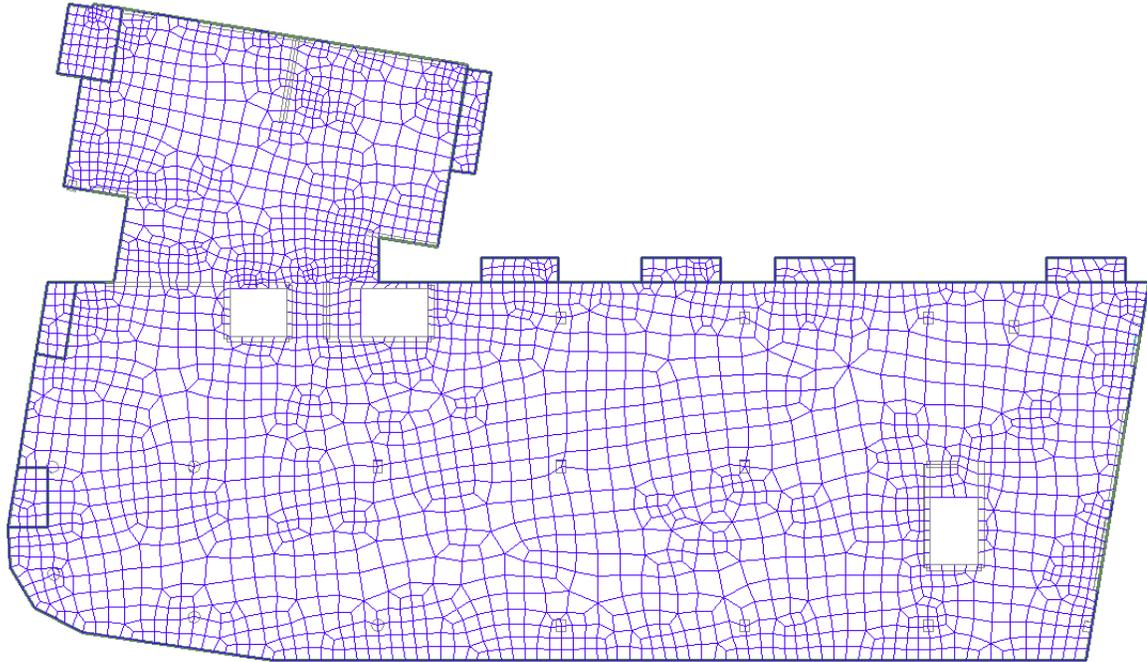


FIGURE 5 DISCRETIZATION OF THE TYPICAL FLOOR SLAB FOR FINITE ELEMENT ANALYSIS (FLOOR-PRO)

#### EXAMPLE

For the same geometry and parameters of examples 1 and 2, using a finite element program determine the deflection at center of the panel identified in Fig. EX1-1.

ADAPT-FLOOR Pro<sup>4</sup> program was used to model the slab and obtain a solution. The distribution of deflection for the given load is shown in Fig. EX3-1. The maximum deflection at the center of the panel under consideration is reported as 0.54 in.

<sup>4</sup> ADAPT-Floor Pro is a finite element program for analysis and design of conventionally reinforced or post-tensioned floor systems. [www.adaptsoft.com](http://www.adaptsoft.com)

uncracked\_DL+LL-Deflection: Z-Translation: [ 1 Contour = 0.054 in ];  
 Maximum Value = 7.222e-001 (in) @ [13.671 377.073 9.667]ft;  
 Minimum Value = -8.054e-002 (in) @ [27.694 397.003 9.667]ft;

Unit :  
in

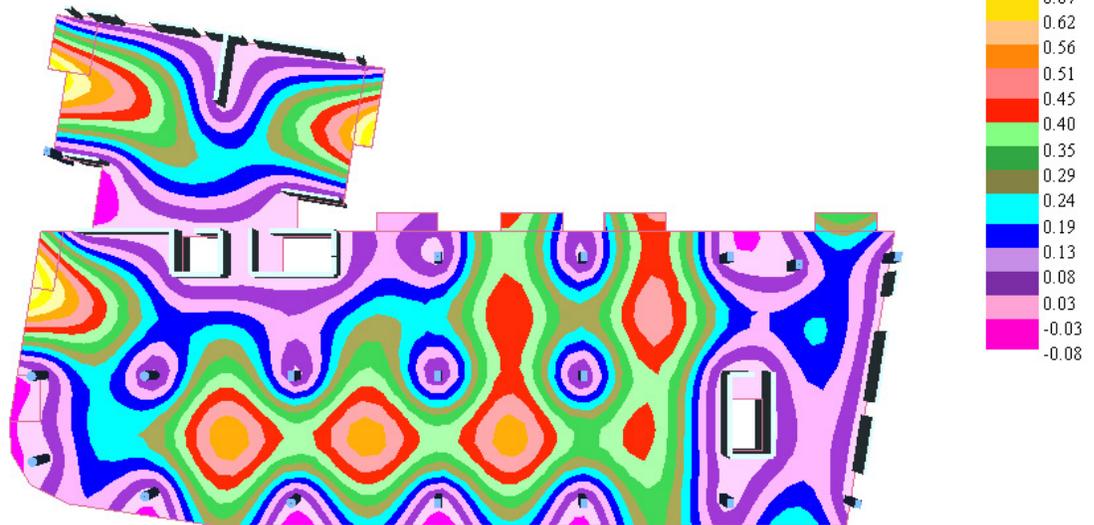


FIGURE EX3-1 DEFLECTION CONTOUR OF THE FLOOR SYSTEM UNDER THE COMBINED ACTION OF DEAD AND LIVE LOADS

### Using Finite Element Method With Due Allowance for Cracking

Formulation of finite elements with allowance for cracking is somewhat complex. The complexity arises from the fact that cracking and reduction in stiffness depend on the presence, amount and orientation of reinforcement. Before a solution is obtained, the reinforcement detailing of a floor system must be fully known, since the loss of stiffness in each finite element cell depends on the availability and exact location of the reinforcement in that cell.

The following briefly describes the steps for a finite element deflection calculation, with allowance for cracking.

1. Using the geometry, boundary conditions, material properties, and the load combination for which the deflection is sought, the program discretizes the structure, sets up the system stiffness matrix of the structure based on gross moment of inertia ( $I_g$ ), and obtains the distribution of moments ( $M_a$ ) over the entire structure.
2. If required the program performs a design check, using a building code, and adds the required reinforcement to the floor system for the specified loads.
3. The Program scans the entire floor system to detect the reinforcement available in the beams and the slab regions. The available reinforcement is either determined by the Program prior to the initiation of deflection calculation, or is a combination of program calculated and user defined/edited reinforcement.

Once the deflection calculation is initiated, the available reinforcement remains unchanged. The reinforcement can be in one or more of the following forms, with no restriction on the orientation, length, or the position of each reinforcement within the floor system.

- a - User defined one or more top and bottom reinforcement mesh;
- b -. User defined grouped or distributed reinforcement bars at top and/or bottom of

- slab and beams;
- c - Reinforcement calculated and reported by the program for minimum requirements of the code, strength check, initial condition, or other code related criteria; and
- d - Post-tensioning tendons defined by the user, each with its own location and force.

4. The program matches the calculated moment ( $M_a$ ) of each finite element cell with the existing reinforcement in that cell. Using the parameters given below, the Program calculates the effective moment of inertia for each cell:

- a. Finite element cell thickness (to obtain uncracked second moment of area);
- b. Available reinforcement associated with each cell (nonprestressed and prestressed), with recognition of orientation and height of each individual reinforcement;
- c. Cracking moment of inertia associated with each cell in each direction ( $I_{cr}$ );
- d. Cracking moment associated with each cell ( $M_{cr}$ ); and
- e. Applied moment ( $M_a$  calculated in 2).

5. Having determined the effective second moment of area of each finite element cell in each of the principal directions, the Program re-constructs the stiffness matrix of each cell.

6. The Program re-assembles the system stiffness matrix and solves for deflections. At this stage the solution given from the first iteration is a conservative estimate for the floor deflection with cracking. For practical engineering design, it is recommended to stop the computations at this stage.

For a more accurate solution the newly calculated deflection can be compared with that of a previous iteration. If the change in maximum deflection is more than a pre-defined tolerance, the Program will go into another iteration starting from step (2). In this scenario, the iterations are continued, until the solution converges to within the pre-defined tolerance.

The Program accounts for loss of bending stiffness in beams and slab regions and its combinations..

#### EXAMPLE

Using finite elements determine the deflection of the panel identified in Fig. EX1 for the loads and conditions described in Example 1.

Calculate the deflection for the combination of dead and live loads.

Use ACI318-08 to determine the reinforcement necessary for both the in-service and strength requirements of the code.

Use the calculated minimum reinforcement of the code to determine the cracked deflection. No other reinforcement is added.

Using the above requirements, the cracked deflection of the floor system is calculated and illustrated as a contour image in Fig. EX4-1. The deflection at the center of the panel under consideration is 0.70 in. (17.78 mm) compared to 0.54 in. (13.72 mm) for the uncracked slab. The cracked deflection can be reduced by adding reinforcement at the locations of crack formation in addition to the minimum requirements of the code already included in the analysis.

cracked\_DL+LL-Deflection: Z-Translation:[ 1 Contour = 0.071 in ];  
Maximum Value = 9.819e-001 (in) @ [13.671 377.073 9.667]ft;  
Minimum Value = -8.723e-002 (in) @ [27.694 397.003 9.667]ft;



FIGURE EX4-1 DEFLECTION CONTOUR OF SLAB WITH CRACKING

The locations of crack formation and the extent of cracking are illustrated in Figs. EX4-2 and EX4-3. At each location, the reduction in effective moment of inertia is based on the calculated moment at that location and the amount, position and orientation of reinforcement at the same location. The largest loss of stiffness occurs over the columns and the support lines joining the columns. The maximum loss of stiffness is 69% reducing the effective moment of inertia to 31% of its uncracked value.

Cracked\_DL+LL-Deflection: Reduced Stiffness Ratio About XX:[ 1 Contour = 0.046 ];  
 Maximum Value = 1.000e+000 @ [163.790 326.820 9.667]ft;  
 Minimum Value = 3.148e-001 @ [129.100 356.008 9.667]ft;

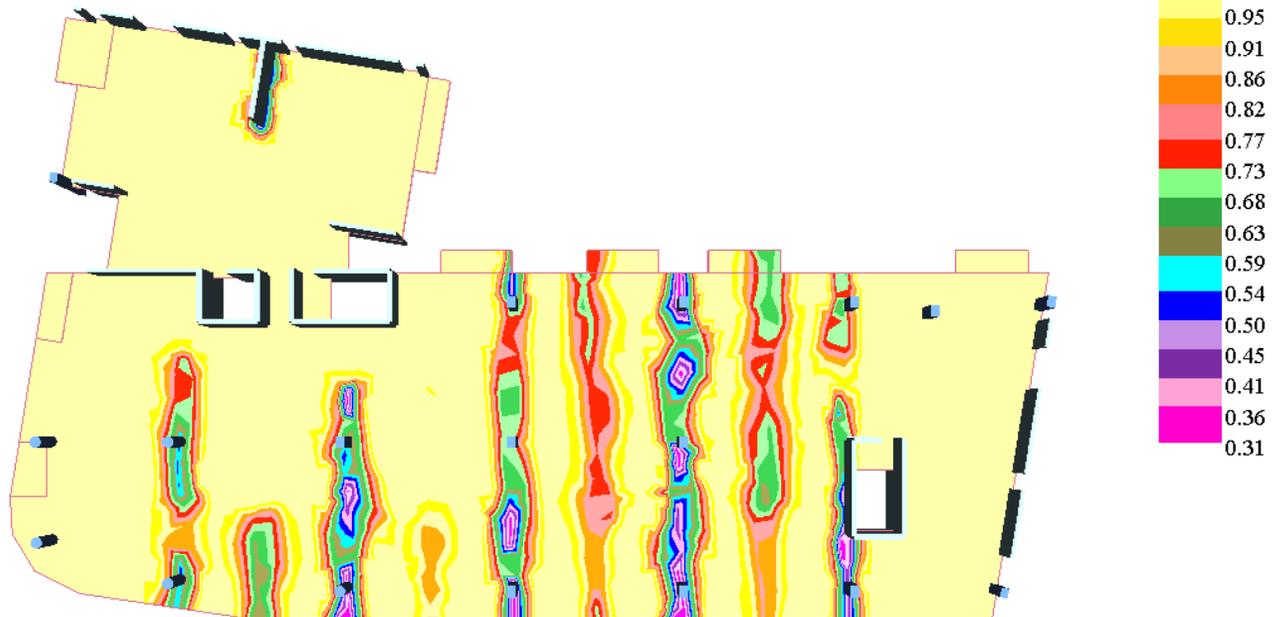


FIGURE EX4-2 EXTENT OF CRACKING SHOWN THROUGH REDUCTION IN EFFECTIVE MOMENT OF INERTIA I<sub>e</sub> ABOUT Y-Y AXIS

Cracked\_DL+LL-Deflection: Reduced Stiffness Ratio About YY:[ 1 Contour = 0.045 ];  
 Maximum Value = 1.000e+000 @ [163.790 326.820 9.667]ft;  
 Minimum Value = 3.297e-001 @ [129.100 356.008 9.667]ft;

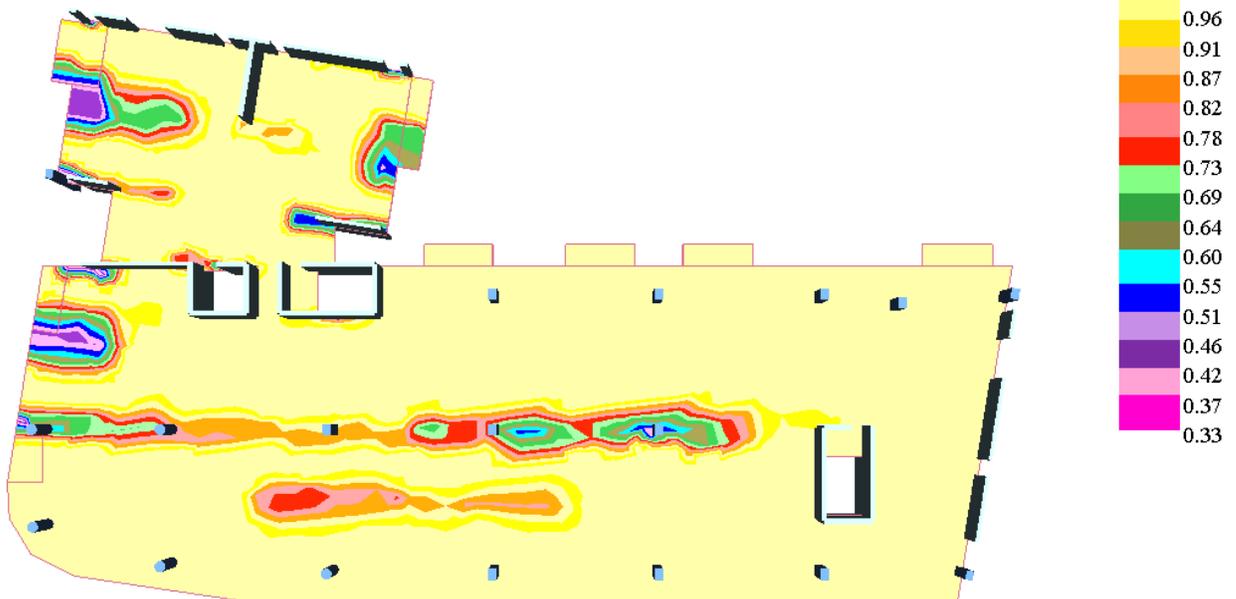


FIGURE EX4-3 EXTENT OF CRACKING SHOWN THROUGH REDUCTION IN EFFECTIVE MOMENT OF INERTIA I<sub>e</sub> ABOUT X-X AXIS

**Deflection of Post-Tensioned Floor Systems**

Two-way post-tensioned floor systems designed to ACI-318 provisions and the PTI-recommended slab-to-depth ratios (Table 4), either do not crack under service condition, or crack to an extent that does not invalidate calculations based on gross cross-sectional geometry and linear elastic theory. This is because, unlike other major non-US codes, the allowable tensile stresses in ACI are relatively low.

The preceding observation does not hold true for post-tensioned one-way slab and beams, where ACI-318 permits designs based on post-cracking regime. For such post-tensioned floor systems, designers must include allowance for cracking in their designs.

**COMPARISON OF DEFLECTION CALCULATION METHODS**

As illustrated in the design examples, the calculated deflections design engineers use from the ??? currently available procedures for conventionally reinforced concrete vary greatly. Table 6 lists the outcome of the various methods. Note that for the typical floor system selected, the difference between the various methods can be as much as three times. Finite Element Method with due allowance for crack formation gives the largest deflection. The strip method with no allowance for cracking produces the smallest value.

**TABLE 6 DEFLECTION VALUES AT CENTER OF PANEL OF THE NUMERICAL**

	<b>Calculation Method</b>	<b>Deflection in(mm)</b>	<b>Normalized Deflection</b>
1	Closed form formulas	0.480(12.19)	69 %
2	ACI318 – Simplified method	0.528(13.42)	75 %
3	Strip method (uncracked)	0.462(11.74)	66 %
4	Strip method with cracking and numerical integration	0.470(11.94)	67 %
5	Finite Element Method (FEM) No allowance for cracking	0.540(13.72)	77 %
6	Finite Element Method (FEM) With allowance for cracking	0.700(17.78)	100 %

## LONG-TERM DEFLECTIONS

A concrete member's deformation changes with time due to shrinkage and creep. Shrinkage of concrete is due to loss of moisture. Creep is increase in displacement under stress. Under constant loading, such as selfweight, the effect of creep diminishes with time. Likewise, under normal conditions, with loss of moisture, the effect of deformation due to shrinkage diminishes. Restraint of supports to free shortening of a slab due to shrinkage or creep can lead to cracking of slabs and thereby an increase in deflection due to gravity loads.

While it is practical to determine the increase in instantaneous deflection of a floor system due to creep and shrinkage at different time intervals, the common practice for residential and commercial buildings is to estimate the long-term deflection due to ultimate effects of creep and shrinkage.

### **Shrinkage**

It is the long-term shrinkage due to loss of moisture through the entire volume of concrete that impacts a slab's deformation. Plastic shrinkage that takes place within the first few hours of placing of concrete does not play a significant role in slab's deflection and its impact in long-term deflection is not considered.

Long-term shrinkage results in shortening of a member. On its own, long-term shrinkage does not result in vertical displacement of a floor system. It is the presence of non-symmetrical reinforcement within the depth of a slab that curls it (warping) toward the face with less or no reinforcement. The slab curling is affine to its deflection due to selfweight, and hence results in a magnification of slab's natural deflection.

It is important to note that, deflection due to shrinkage alone is independent of the natural deflection of slab. It neither depends on the direction of deflection due to applied loads, nor the magnitude. The shrinkage deflection depends primarily on the amount and position of reinforcement in slab.

A corollary impact of shrinkage is crack formation due to restraint of the supports. This is further discussed in connection with the restraint of supports. It is the crack formation due to shrinkage that increases deflection under gravity loads.

Shrinkage takes place over a time period extending beyond a year. While the amount of shrinkage and its impact on deflection can be calculated at shorter intervals, the common practice is to estimate the long-term deflection due to the ultimate shrinkage value.

Shrinkage values can vary from zero, when concrete is fully immersed in water to 800 micro strain. Typical ultimate shrinkage values are between 400 to 500 micro strain.

### **Creep**

Creep is stress related. It is a continued magnification of the spontaneous displacement of a member with reduced rate of creep with time. Values of creep vary from 1.5 to 4. Typical ultimate creep values for commercial and building structures are between 2 to 3.

### **Restraint of Supports**

Restraint of supports, such as walls and columns to free movement of a slab due to shrinkage can lead to tensile stresses in the slab and early cracking under applied loads. Early cracking will reduce the stiffness of the slab and increase its deflection.

**Multiplier Factors for Long-Term Deflections**

For design purposes, the long-term deflection of a floor system due to creep and shrinkage can be expressed as a multiplier to its instantaneous deflection.

Long-term deflection due to sustained load:

$$\Delta_l = C * \Delta_i \tag{8}$$

Where

- $\Delta_l$  = long-term deflection;
- $\Delta_i$  = instantaneous deflection; and
- C = multiplier.

ACI-318 suggests the multiplier factor shown in Fig. 5 to estimate long term deflections due to sustained loads

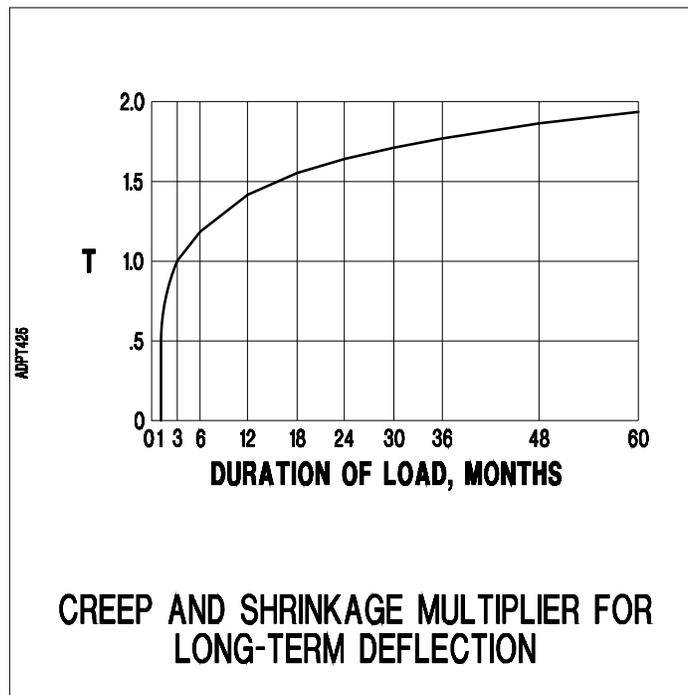


FIGURE 5 MULTIPLIER FOR LONG-TERM DEFLECTION

The multiplier can be reduced, if compression reinforcement is present. The factor ( $\lambda$ ) for the reduction of the multiplier is given by:

$$\lambda = C / (1 + 50\rho') \tag{9}$$

Where  $\rho'$  is the value of percentage of compression rebar at mid-span for simple and continuous members and at support for cantilevers.

ACI's recommended multipliers account for the cracking of slab. Hence, they are intended to be applied to cracked deflections. Several Investigators recommend long-term multiplier coefficients for deflections based on gross cross-sectional area. These coefficients are higher

than the ACI-318 multiplier (Fig. 5). Table 7 lists the recommended values of multipliers for non-prestressed slabs.

$$\Delta_l = ( 1 + \lambda_c + \lambda_{sh} ) * \Delta_i \tag{10}$$

Where

$\lambda_c$  = creep multiplier;

$\lambda_{sh}$  = shrinkage multiplier;

Or, simply  $\Delta_l = C * \Delta_i$  , where  $C = ( 1 + \lambda_c + \lambda_{sh} )$

**TABLE 7 MULTIPLIERS FOR LONG-TERM DEFLECTIONS**

<b>Source</b>	<b>Immediate deflection</b>	<b>Creep <math>\lambda_c</math></b>	<b>Shrinkage <math>\lambda_{sh}</math></b>	<b>Total C</b>
Sbarounis(1984)	1.0	2.8	1.2	5.0
Branson(1977)	1.0	2.0	1.0	4.0
Graham and Scanlon (1986b)	1.0	2.0	2.0	5.0
ACI-318	1.0	2.0		3.0

Based on the author’s observation and experience, it is recommended that structures built in California use the following values:

For conventionally reinforced floor systems                    C = 4  
 For post-tensioned floor systems                                    C = 3

**LOAD COMBINATIONS**

The load combination proposed for evaluating the deflection of a floor system depends on the objective of the floors evaluation. The following describe several common scenarios.

**Total Long-Term Displacement From Removal of Forms**

$$(1.0*SW + 1.0*SDL + 1.0*PT + 0.3*LL)* C$$

Where

- SW = selfweight;
- SDL = superimposed dead load, (floor cover and partitions);
- PT = post-tensioning; and
- LL = design live load.

The above load combination is conservative as it assumes the application of superimposed loads as well as the application of sustained live load of the structure to take place at the time of removal of the supports below the cast floor. The factor 0.3 suggested for live load is for

“sustained” load combination. The significance of the above load combination is that it provides a measure for the total deflection from the position of the forms at the time of concrete casting. Its magnitude must be evaluated for aesthetics and drainage of surface water, if applicable. It is used for checking the deflection of parking structure decks or roofs, where the floor is placed in service in its as-cast condition.

**Load Combination for Code Checks**

For the acceptability of a floor deflection in connection with the code specified maximum values listed in Table 1, the following two load combinations apply.

$$1.0*LL$$

$$C1*C*(SW + SDL + PT) + 0.3*C2*C*LL + 0.7*LL$$

Where C1 is the fraction of long-term deflection coefficient related to the balance of long-term deflection subsequent to construction installation likely to be damaged by deflection of slab. Partitions and other fixtures are generally installed when more than one-half of long-term deflection has taken place. As a result, C1 is generally less than 50% of the long-term deflection multiplier, assuming that the superimposed dead load and partitions are not installed before 40 days from date of casting the floor (Fig. 6). C2 relates to the time, when construction is complete and in-service live load applied. This is generally less than 20% of the long-term multiplier. Figure 6 can be used as a guideline for values of C1 and C2. For example, if the in-service live load of a structure is put in place six months subsequent to casting the floor, the value of C2 will be approximately 0.25 (value associated with 180 days in Fig. 6).

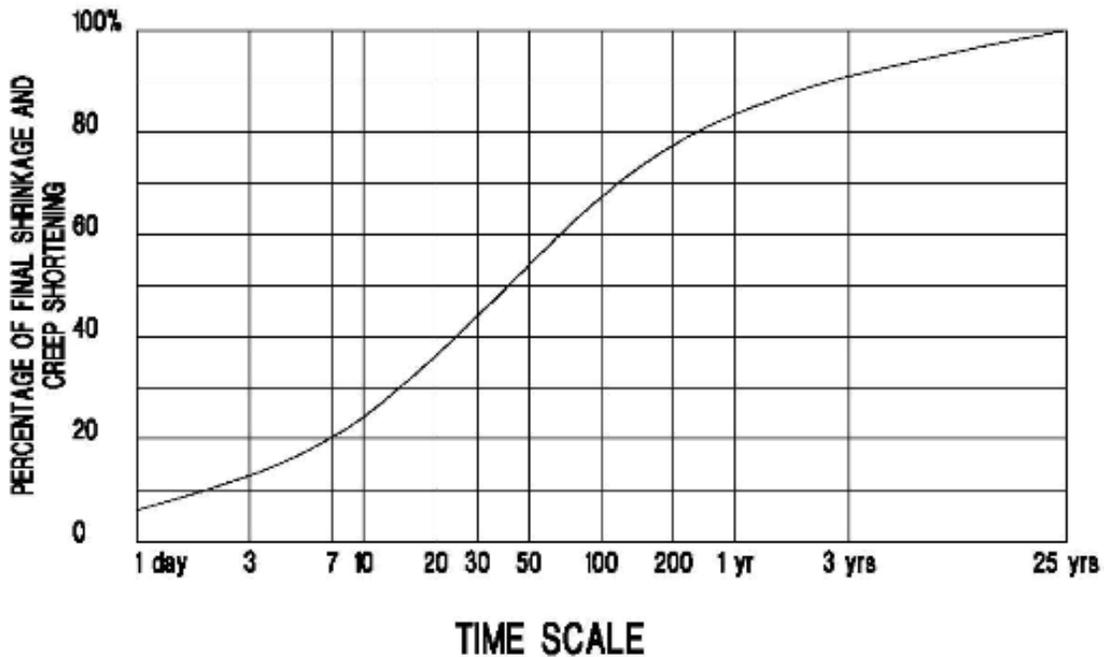


FIGURE 6 LONG-TERM SHORTENING OF CONCRETE MEMBERS DUE TO CREEP AND SHRINKAGE WITH TIME

LIVE LOAD DEFLECTION

Even when using linear elastic theory to calculate a floor system's deflection, cracking will result in a non-linear response. For the same load, the deflection of a slab depends on the extent of cracking prior to the application of the load. Therefore, when calculating the deflection due to the instantaneous application of live load, one must use the following procedure:

$$\text{Deflection due to LL} = (\text{deflection due to DL+LL}) - (\text{deflection due to DL})$$

The above accounts for loss of stiffness due to dead load prior to the application of live load.

## APPENDIX A

### CHARACTERISTICS OF PARISSA APARTMENTS TYPICAL FLOOR

#### Geometry

Slab thickness and support dimensions (see plan)

#### Concrete

$f'_c$ (28 day cylinder strength)	= 5000 psi (34.47 MPa)
Wc (unit weight)	= 150 pcf (2403 kg/m <sup>3</sup> )
Ec (modulus of elasticity at 28 days)	= 4,287 ksi (29558 MPa)

#### Non-Prestressed Reinforcement

Yield stress	= 60 ksi (400 MPa)
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