

VIBRATION DESIGN OF CONCRETE FLOORS FOR SERVICEABILITY¹

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This Technical Note covers the design of concrete floor systems for vibration, with an emphasis on simple and expeditious first estimates for a floor's vibration response. The objective is to determine whether a floor meets the serviceability requirements for vibration using conservative values, or whether a more detailed analysis is warranted. The Technical Note includes several numerical examples to illustrate the application of the procedures presented.

SCOPE

In general, the vibration response of a concrete floors is attributed to:

- ❖ Consequence of foot drop in residential and commercial floors;
- ❖ Rhythmic vibration, such as in dance and sport events
- ❖ Vibration in laboratories; manufacturing facilities
- ❖ Vibrations due to vehicular traffic outside a facility
- ❖ Vibrations due to operation of machinery
- ❖ Transient impulse due to earthquake, wind or other impact loads

The focus of this Technical Note is design for vibration in residential and commercial buildings, due to foot drop of walking occupants.

OVERVIEW

The analytical background of vibrations is well developed and understood. Detailed and rigorous computational tools are available to analyze vibration and response of both simple and complex structures. The difficulty in vibration design is the poor correlation between the outcome of computations at the design stage, and the response of the floor constructed accordingly. In addition to the uncertainties inherent in material properties, damping characteristics, and boundary conditions, the level of vibrations perceived by individuals, the vibration that is considered objectionable, and the force and frequency of foot drop are all highly subjective and prone to large variations.

The following factors must be identified at initiation of design:

- ❖ The vibration source
- ❖ The vibration transmission path (mass, stiffness, damping)
- ❖ What vibration can be perceived and is objectionable
- ❖ How to determine the vibration characteristics of a floor at design stage
- ❖ How to determine whether or not the vibration response is acceptable

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Two methods are frequently employed.

- ❖ “Frequency Tuning Method” sets the natural frequency of a floor system above frequencies susceptible to resonating when excited by the lower harmonic of walking forces.
- ❖ “Response Calculation Method,” a performance-based method focuses on determining the likely vibration response of a floor under the application of a dynamic force from walking.

This Technical Note presents a simplified procedure based on the “Response Calculation Method.” Due to the variable nature of the parameters that determine the response of a floor system, such as damping, this procedure is valuable for a first design estimate. Damping cannot be calculated as such, and must always be assessed based on experience with floors of similar construction.

VIBRATION SOURCE

This Technical Note considers vibration source from foot drop of walking individuals. Walking rates (foot drops) above 2.5 Hz are uncommon so this is a reasonable upper limit for the design of corridors and large circulation areas. For an open plan office, an upper limit of 2.1 Hz is acceptable. For low height partitioned office spaces and labs, 1.8 Hz is more appropriate [TR43, 2005]. In the absence of more detailed information, a walking frequency of 2 Hz is recommended.

The force of vibration depends on the weight and walking style of the individual and is a fraction of the individual’s weight. Figure 1 offers an approximate guideline adapted from reference [TR43, 2005] for determining the fraction of a walking person’s weight, expressed in terms of Dynamic Load Factor (DLF) as a function of a person’s walking frequency.

TRANSMISSION PATH OF VIBRATION

The parameters governing the vibration response of a floor system are its mass; modulus of elasticity; damping; extent of cracking if any; and post-tensioning.

Mass

The mass to be used in analysis is that of the floor system and its superimposed load. It is expressed as (W/g) , where “W” is the weight of the objects attached to the floor that faithfully follow its displacement and “g” is the gravitational acceleration taken as 32.2 ft/sec^2 (9.81 m/sec^2). Applied forces without mass that do not affect structure’s stiffness are not included in the vibration response of a floor.

Modulus of Elasticity

The elastic modulus for vibration analysis is larger than the static values, in particular when high strength concrete is used. Recommended values are 25% higher than the static modulus.

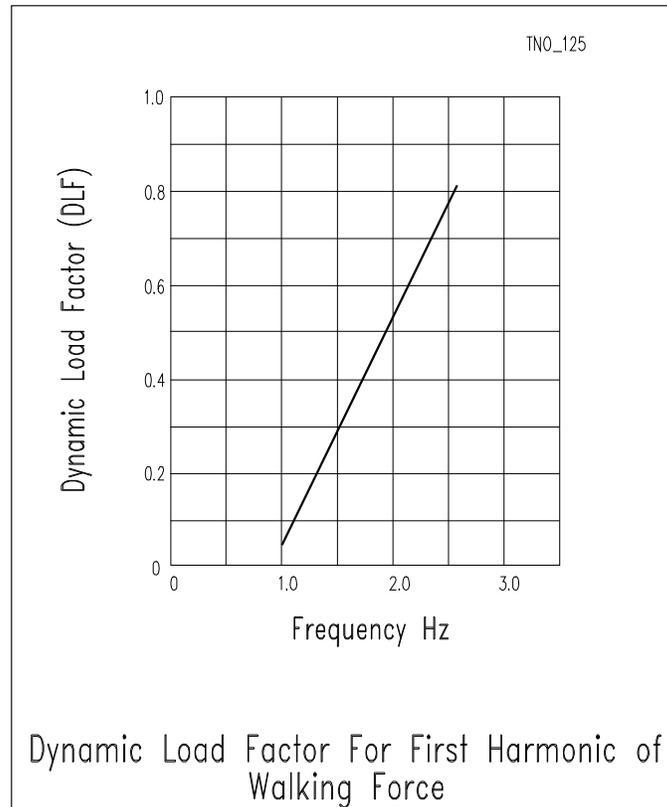


FIGURE 1

Damping

Damping has an inherently high variability that is difficult to determine before a floor system is placed in service. The recommended values from reference [Allen, D.E., and Murray, T. M., 1993] vary from 2-3% for bare concrete floors to 5-8% with full height partitions. Damping factors suggested in the same reference are listed in Table 1.

TABLE 1 RECOMMENDED DAMPING FACTORS FOR VARIOIUS OCCUPANCIES

Occupancy	Damping factor β
Bare concrete floor	0.02
Furnished, low partition	0.03
Furnished, full height partition	0.05
Shopping malls	0.02

Extent of Cracking

Cracking reduces floor stiffness and, consequently, lowers its natural frequency. For conventionally reinforced concrete it is important to allow for cracking. Otherwise, the results are likely to be on the unconservative side. For conventionally reinforced flat slab construction with span to depth ratio of 30 or larger, a 30% reduction in stiffness is reasonable. For post-tensioned floors designed according to IBC [IBC, 2006], allowable tensile stresses are low so reduction in stiffness is not necessary. Designs

based on the European code EC2 and most other major non-US codes permit a greater extent of cracking in post-tensioned floor systems, where a reduction in stiffness for vibration design may be necessary.

Post-Tensioning

Post-tensioning imparts axial compression. When axial compression results from an externally applied force (see Fig 2-a), the displacement of the compressed member is accompanied by an applied moment ($P \cdot \Delta$). This reduces the flexural stiffness of the member and consequently a reduction in natural frequency. The observation expressed for externally applied forces does not apply for post-tensioned tendons, be they internal or external, provided the displacement of tendon is locked to that of the member (Fig. 2- c and d). Hence, the vibration design of post-tensioned members is the same as their conventionally reinforced counterparts, except that cracking is inhibited or is less in post-tensioned members.

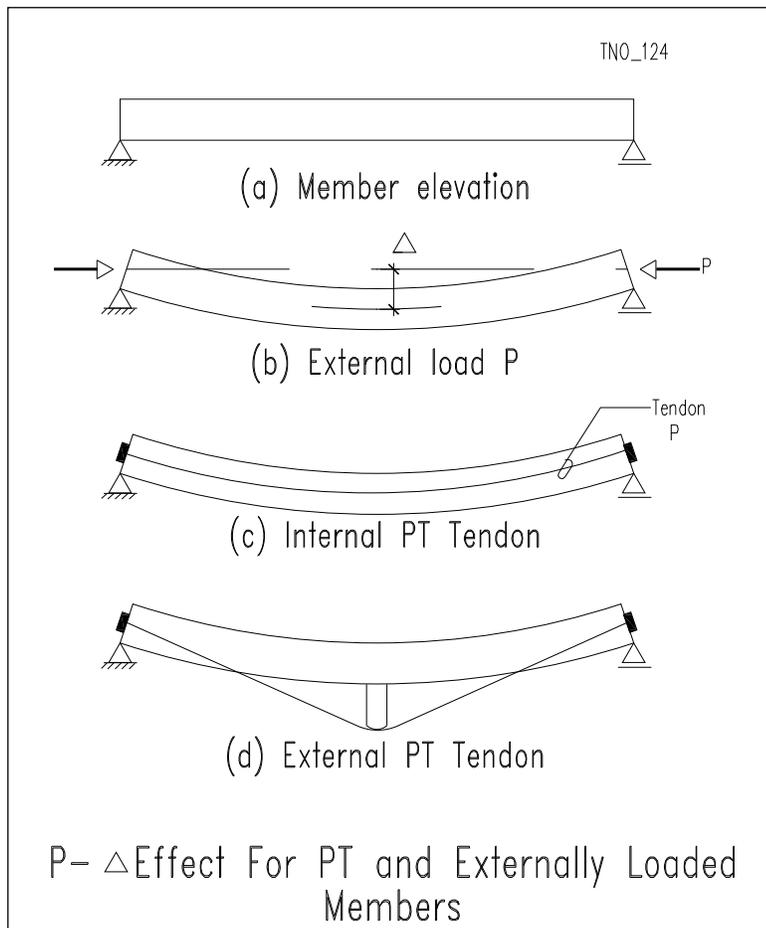


FIGURE 2

PERCEPTION OF VIBRATION

Perception of vibration, and whether or not it is annoying or objectionable is highly subjective and varies from reference to reference. Canadian Steel Code (CAN3-S16.1-M89) defines the “perceptibility” for foot drop according to Fig.3 for various levels of floor damping. The Applied Technology Council

[ATC, 1999] addresses the same issue and recommends the threshold of human sensitivity to vertical vibration as shown in Fig. 4. Other references state somewhat different values. In most cases the perceptibility is related to the response acceleration of the floor system for different natural frequencies of the floor. The common consensus among the investigators is that humans are most sensitive to vibration for frequencies between 4 to 8 Hz. Larger acceleration values can be tolerated at higher or lower frequencies.

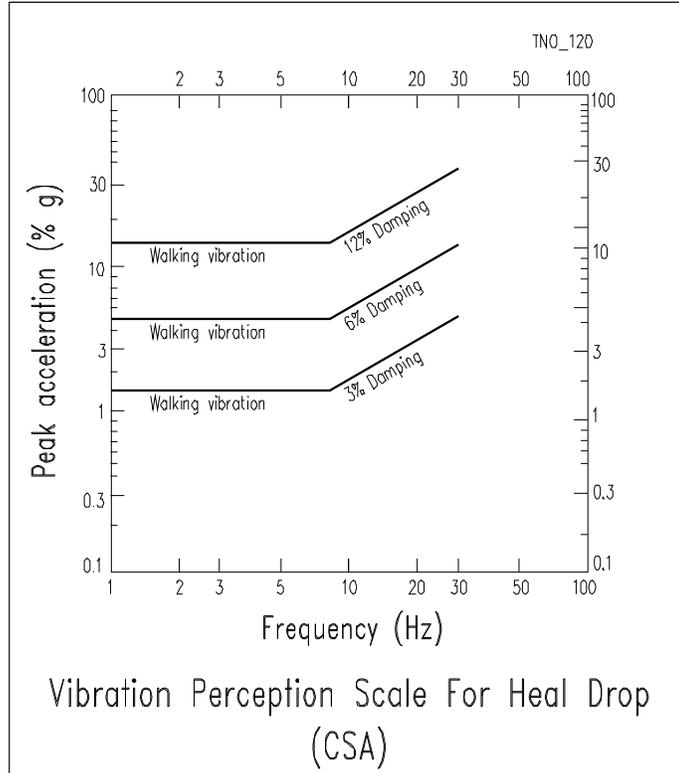


FIGURE 3

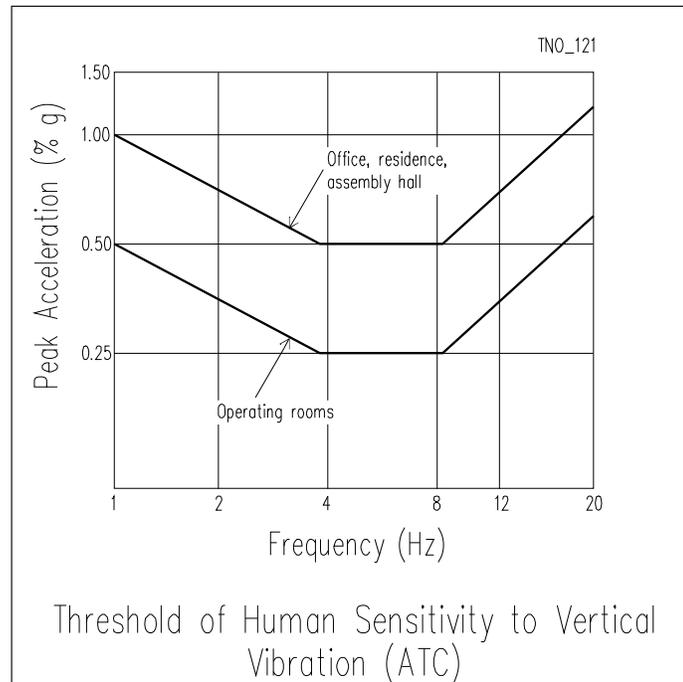


FIGURE 4

DETERMINATION OF VIBRATION CHARACTERISTICS OF A FLOOR

Determining vibration characteristics requires finding (i) the natural frequency of a floor system and (ii) the associated peak acceleration. Several methods are available, including commercial programs, to determine the natural frequency of a floor. The simplified method described herein is based on closed formulas for the first mode natural frequency of uniformly thick rectangular slabs with different boundary conditions.

Observe Fig. 5-a. Note that for interior spans the displacement shape under selfweight is analogous to that of a single panel fixed at its supports (part d of figure 5). However, the first mode of vibration as shown in part c of the figure implies that the vibration response of the floor is more affine to that of a single panel simply supported along its four sides (part a). For this reason, for panels bound by similarly sized spans, it is recommended to use a simply supported boundary condition along the four sides of the panel. More specifically, the recommended conditions is rigid supports, but rotationally free. This holds true for column supported panels too.

Where columns are organized on a regular orthogonal grid, the first natural frequency of a two-way slab is likely to be in the form of a one-way slab deflecting in a cylindrical form. This alternative is described in greater detail in the following.

For panels that are bound by smaller spans, in the limit as the size of the adjacent panels reduce, the vibration mode will be analogous to a rotationally fixed condition, as indicated in part (b) of figure 5. Table 2 [Bares, 1971] provides values for both extremes of simply supported and fully fixed. Case 1 of the table refers to a central panel bound by similar panels on each side. The first natural frequency “f” is given by:

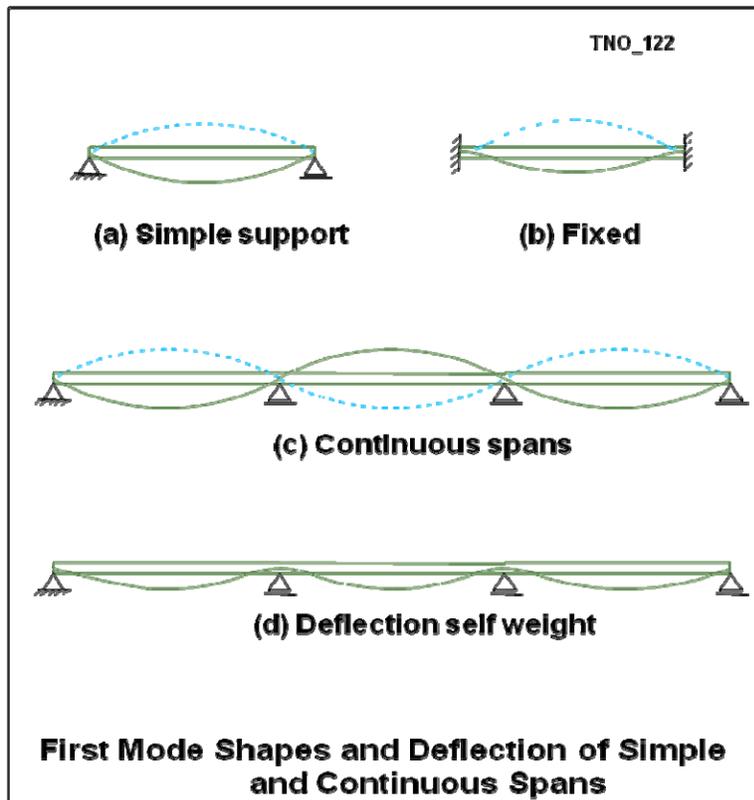


FIGURE 5

The parameters to use with Table 2 for the computation of the first natural frequency of rectangular slab panels are explained next.

$$f = \frac{c}{a^2} \phi \quad (1)$$

where

$$c = \sqrt{\frac{Eh^3}{12(1-\nu^2)}} \times \frac{g}{q} \quad (2)$$

- f = first natural frequency [Hz];
- a = span length in X-direction;
- E = dynamic modulus of elasticity [1.25 static E in psi; MPa];
- h = slab thickness [in; mm];
- ν = Poisson's ratio [0.2];
- g = gravitational acceleration [32.2 ft/sec² ; 9810 mm/sec²]; and
- q = weight per unit surface area of the slab.

Alternatively, the following relationship can be used to calculate the first natural frequency [Szilard, 1974]. First natural frequency “f” is given by

$$f = \frac{\omega}{2\pi} \quad (3)$$

Where,

$$\omega = \pi^2 \left(\frac{1}{a^2} + \frac{1}{b^2} \right) \sqrt{\frac{D}{m}} \quad (4)$$

$$\sqrt{\frac{D}{m}} = \sqrt{\frac{Eh^3}{12(1-\nu^2)}} \times \frac{g}{q} \quad (5)$$

For column-supported structures, a two-way action resists the applied transverse loads, and the first mode of vibration can be that of a one-way strip. Consider the typical floor of a multi-story building shown in Fig. 6.

The first likely natural frequency of this floor can be in the shape shown in Fig. 7 for the lower part of the floor.

For a long plate strip resting freely on two opposite sides the first natural frequency from the relationships 4 and 5 reduces to the following:

$$f = (1.57 / b^2)^* \sqrt{(D/m)} \quad (6)$$

Where, $\sqrt{(D/m)}$ is given by expression (5):

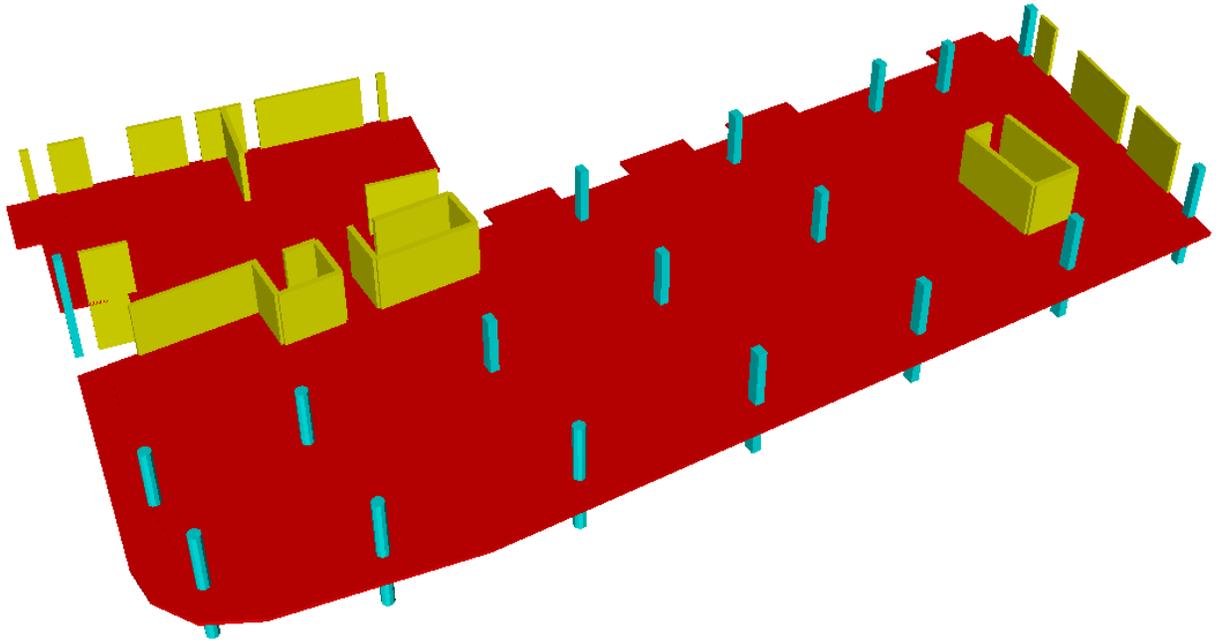


FIGURE 6 VIEW OF A TYPICAL FLOOR OF A MULTISTORY BUILDING

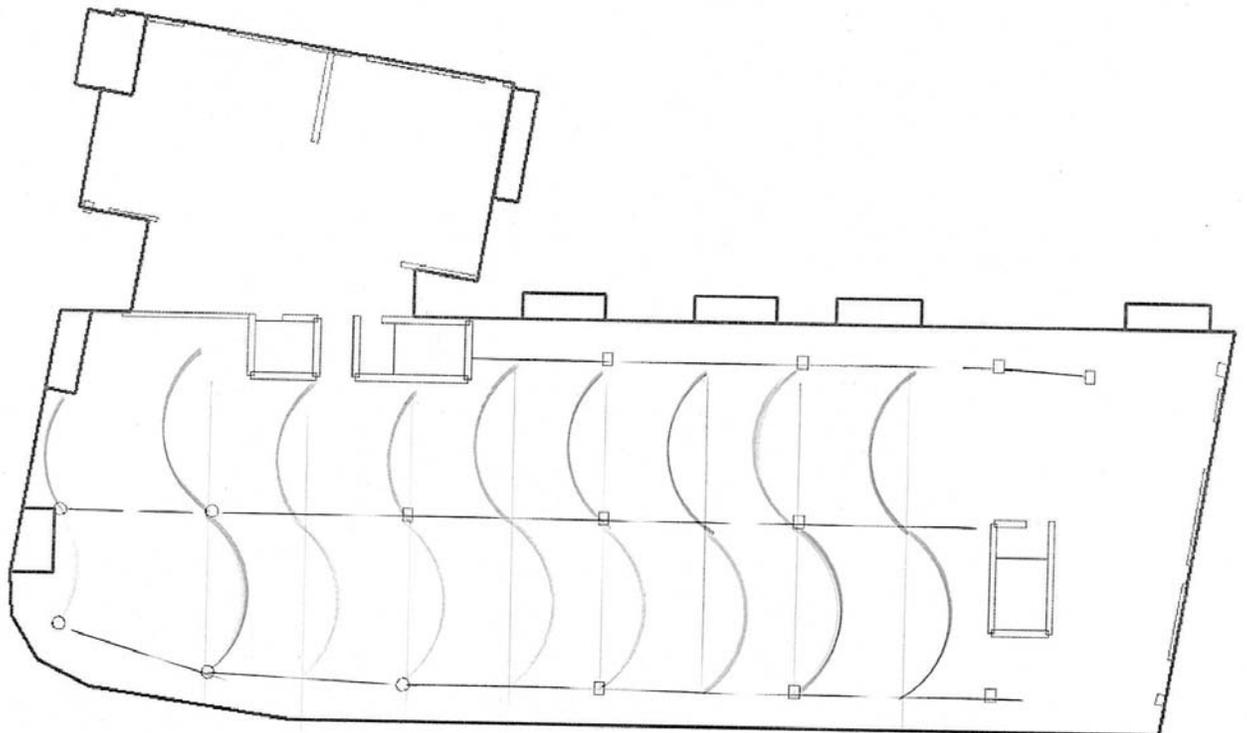


FIGURE 7 POSSIBLE LOWEST NATURAL FREQUENCY SHAPE

PEAK ACCELERATION AND ACCEPTABILITY OF VIBRATION

To evaluate the vibration of a floor system, designers must determine the floor's peak acceleration response from foot drop, since the acceleration response is one of the two prime parameters in perception of vibration. Peak acceleration is obtained from the first natural frequency of a floor. [ATC, 1999; AISC/CISC 1997] recommends the following relationship:

$$\frac{a_p}{g} \leq \frac{P_0 e^{-0.35f_n}}{\beta W} \quad (7)$$

where

- a_p = peak acceleration;
- g = gravitational acceleration [32.2 ft/sec²; 9.81 m/sec²];
- P_0 = constant force representing the walking force;
- β = modal damping ratio, recommended in Table 1;
- W = effective weight of the panel and the superimposed dead load; and
- f_n = first natural frequency.

The calculated response acceleration is compared with the minimum acceptable value given by equation 8 [walking [Allen, D.E., and Murray, T.M., 1993]] and the levels per perceptibility (Fig. 4).

Quoting from [Mast, 2001] people are most sensitive to vibration when engaged in sedentary activity while seated or lying. Much more is tolerated by people who are standing, walking, or active in other ways. The following empirical formula, based on resonant effects of walking, has been developed to determine the minimum natural frequency of a floor system needed to prevent disturbing vibration caused by walking [Allen, D.E., and Murray, T.M., 1993]

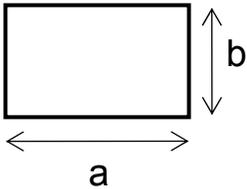
$$f_n \geq 2.86 \ln \left(\frac{K}{\beta W} \right) \quad (8)$$

where

- K = a constant, given in [Table 3];
- β = modal damping ratio [Table 2];
- W = weight of area of floor panel affected by the point load (heel drop); and
- f_n = minimum frequency.

For the first natural frequency and the peak acceleration calculated the acceptability of the floor for vibration perception is compared to and matched against the suggested values of Fig. 4.

TABLE 2 FIRST NATURAL FREQUENCY CONSTANT Φ FOR RECTANGULAR SLAB PANELS OF UNIFORM THICKNESS

Case	Boundary Conditions	Constant ϕ
1		$\phi = 1.57(1 + \gamma^2)$
2		$\phi = 1.57\sqrt{1 + 2.5\gamma^2 + 5.14\gamma^4}$
3		$\phi = 1.57\sqrt{5.14 + 2.92\gamma^2 + 2.44\gamma^4}$
4		$\phi = 1.57\sqrt{1 + 2.33\gamma^2 + 2.44\gamma^4}$
5		$\phi = 1.57\sqrt{2.44 + 2.72\gamma^2 + 2.44\gamma^4}$
6		$\phi = 1.57\sqrt{5.14 + 3.13\gamma^2 + 5.14\gamma^4}$

Notes and Legends:

-  rigidly supported, rotationally free
-  rigidly supported, rotationally fixed
- a** span length in x-direction
- b** span length in y-direction
- γ a/b

TABLE 3 CONSTANT K FOR MINIMUM ACCEPTABLE FREQUENCY

Occupancies	K	
	kips	kN
Offices, residences, assembly halls	13	58
Shopping malls	4.5	20

EXAMPLE

The plan view of a typical level of a multi-story building is shown in Fig. Ex-1. The floor slab is post-tensioned. Evaluate the in-service vibration response of the floor from foot drop and its acceptability. The particulars of the floor are listed below:

- Thickness of the slab = 8 in
- Superimposed dead load = 20 psf
- f_c = 5000 psi
- Poisson's ration (ν) = 0.2
- w_c = 150 pcf
- Superimposed dead load = 20 psf

Select a Critical Panel

For floors of uniform thickness the largest panel is typically selected.

Select Boundary Conditions

The boundary condition 1 listed in Table 2 is the most conservative. It applies to a central panel bound on each side by one or more identical panels. An upper bound to the first natural frequency is the boundary condition 6 noted in the same table. The full fixity simulates connection to thicker core walls.

Find the First Natural Frequency

Using the recommended values for concrete parameters , calculate the first natural frequency of the panel using the formulas of Table 2.

$$g = 32.2 \text{ ft/sec}^2$$

First natural frequency, f_n :

$$f_n = \frac{c}{a^2} \varphi$$

Where,

$$c = \sqrt{\frac{E_{dyn} h^3}{12(1-\nu^2)}} \times \frac{g}{q}$$

$$E_{st} = 33w_c \sqrt{f_c} = (33 * 150^{1.5} \sqrt{5000}) / 1000 = 4287 \text{ ksi}$$

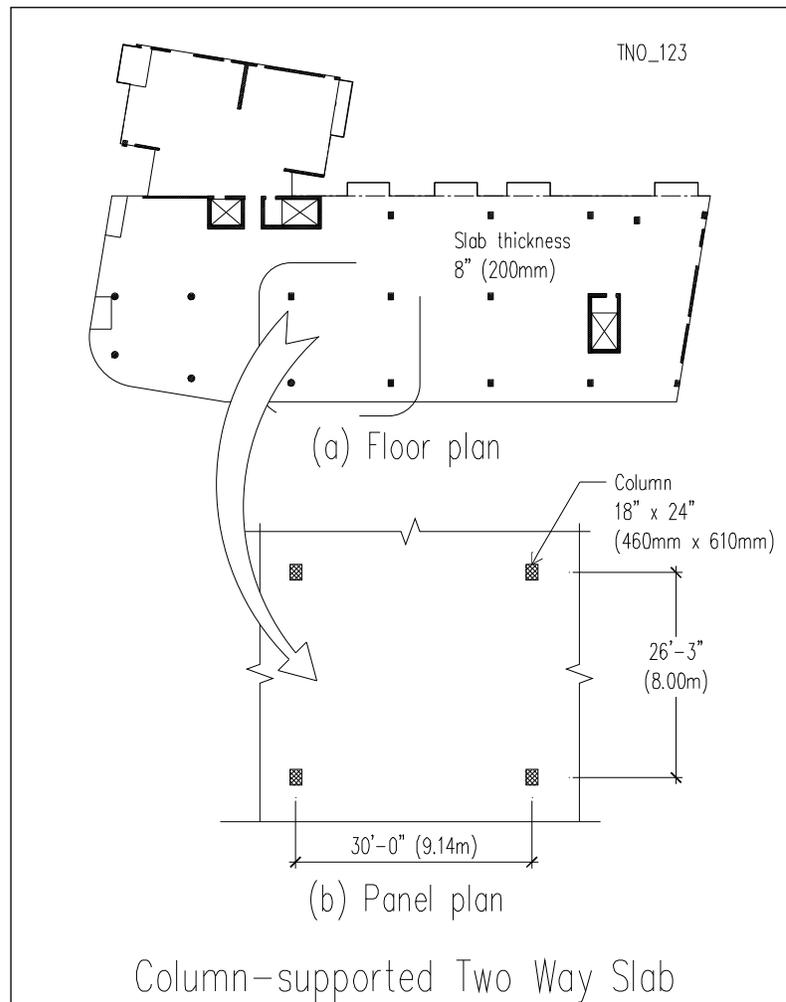


FIGURE EX1

Since there is no reduction in slab stiffness.

Assume $E_{dyn} = 1.2 E_{st} = 1.2 \cdot 4287 = 5144$ ksi

$$q = \text{weight/unit area} = \frac{150 \cdot 8}{12 \cdot 144} + \frac{20}{144} = 0.833 \text{ lb/unit area}$$

$$c = \sqrt{\frac{5144 \cdot 1000 \cdot 8^3}{12(1-0.2^2)}} \times \frac{32.2 \cdot 12}{0.833} = 325,653 \text{ in}^2/\text{sec}$$

$$a = 30 \cdot 12 = 360 \text{ in}$$

$$\begin{aligned} \phi &= 1.57(1+\gamma^2) \\ &= 1.57 \left[1 + \left(\frac{30}{26.25} \right)^2 \right] = 3.62 \end{aligned}$$

$$f_n = \frac{325653}{360^2} * 3.62 = 9.10 \text{ Hz}$$

Determine the Minimum Acceptable Natural Frequency of the Floor

$$f_n \geq 2.86 \ln \left(\frac{K}{\beta W} \right)$$

$$K = 13 \text{ k}$$

$$\beta = 0.03$$

$$W = (30 \times 26.25) \times \left(\frac{8}{12} \times 0.15 + 0.02 \right) = 94.5 \text{ k}$$

$$\text{Minimum acceptable frequency } f_n = 2.86 \ln \left(\frac{13}{0.03 \times 94.5} \right) = 4.36 \text{ Hz}$$

< 9.10 Hz OK

Determine the Peak Response Acceleration

Peak acceleration, a_p :

$$a_p/g = \frac{P_0 e^{-0.35 f_n}}{\beta W} \times g$$

where,

- P_0 = constant force (walking)
- Assume weight = 150 lb
- Walking speed = 2.0 Hz
- DLF = 0.53 (from Fig. 1)
- P_0 = 0.53 * 150 = 79.5 lb

f_n = natural frequency

β = 0.03

W = 94.5 k

$$\frac{a_p}{g} = \frac{79.5 \times e^{-0.35 \times 9.10}}{0.03 \times 94.5 \times 1000} = 0.001 ; \mathbf{0.1 \%}$$

Check the Results for Acceptability

The entry parameters for acceptability of the floor are the first natural frequency (9.1 Hz) and the peak response acceleration relative to gravitational acceleration (0.1 %). The values are checked against ATC chart (Fig. 4)

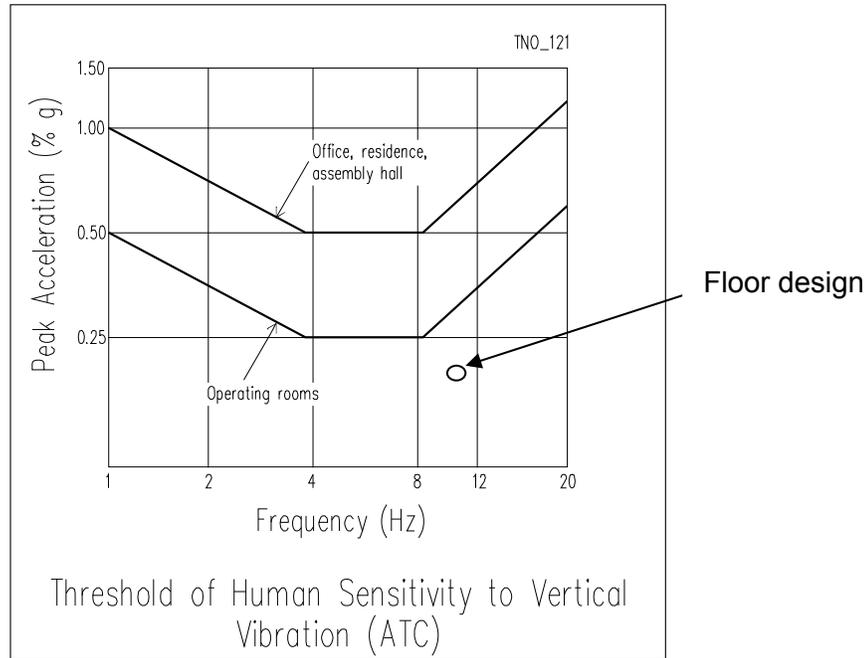


FIGURE EX-2 COMPARISON OF FLOOR RESPONSE WITH THRESHOLD OF HUMAN PERCEPTION

Since the design values are below the threshold of human sensitivity, the design is acceptable.

One-Way Slab Vibration

Using the one-way mode of vibration as illustrated in Fig. 7, with the half waves in direction of 30 ft (9.14 m) span, the first natural frequency of the floor is:

Find the First Natural Frequency

$$f_n = (1.57 / b^2) * \sqrt{D/m} \tag{6}$$

Where, as before $\sqrt{D/m}$ is given by:

$$\sqrt{\frac{D}{m}} = \sqrt{\frac{Eh^3}{12(1-\nu^2)}} \times \frac{g}{q} = \sqrt{\frac{5144 * 1000 * 8^3}{12(1-0.2^2)}} \times \frac{32.2 * 12}{0.833}$$

$$= 325,653 \text{ in}^2/\text{sec}$$

$$f_n = \frac{1.57}{(26.25 \times 12)^2} \times 325653 = 5.15 \text{ Hz}$$

Minimum acceptable frequency, as before, is given by:

$$f_n \geq 2.86 \ln\left(\frac{K}{\beta W}\right)$$

$$f_n = 2.86 \ln\left(\frac{13}{0.03 \times 94.5}\right) = 4.36 \text{ Hz} < 5.15 \text{ Hz} \quad \text{OK}$$

Determine the Peak Response Acceleration

Peak acceleration, a_p :

$$a_p/g = \frac{P_0 e^{-0.35f_n}}{\beta W} \times g$$

where,

- P_0 = constant force (walking)
- Assume weight = 150 lb
- Walking speed = 2.0 Hz
- DLF = 0.53 (from Fig. 1)
- P_0 = 0.53 * 150 = 79.5 lb

- f_n = natural frequency = 5.15 Hz
- β = 0.03
- W = 94.5 k

$$\frac{a_p}{g} = \frac{79.5 \times e^{-0.35 \times 5.15}}{0.03 \times 94.5 \times 1000} = 0.0046 ; \mathbf{0.46 \%}$$

Since the peak response acceleration relative to gravitational acceleration (0.46%) is below the threshold of human sensitivity, the design is acceptable.

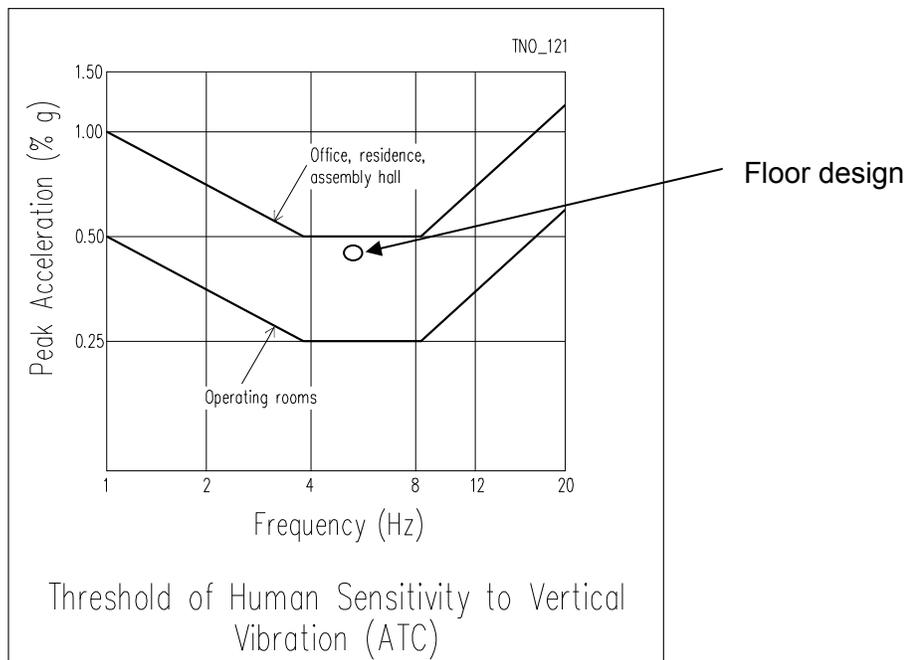


FIGURE EX3 FLOOR'S PEACK ACCELERATION AS A ONE-WAY SLAB AND ITS PERCEPTABILITY

APPLICATION OF VIBRATION-SPECIFIC ANALYTICAL TOOLS

The work described in the preceding sections is based on the traditional methods for the evaluation of vibration characteristics of a floor system, or a specific region of a floor deemed to be sensitive to motion. The trend in structural engineering is to rely on a three dimensional model of a structure that serves other professionals too. While a full model of a floor system has its advantages in portability among the various trades of construction, the complete reliance on a full 3D model can pose drawbacks for structural engineering, in particular when evaluating the vibration of a sensitive floor region. The following explains.

ADAPT-Floor Pro has pioneered a method, whereby from the full 3D model of a building generated for overall design of a structure, a sub-region can be isolated for vibration analysis. The central advantage of this procedure is that it excludes the analytical disturbances of other regions in the vibration characteristics of the floor region of interest. The method is based on selecting a specific region of a floor within the original 3D model, and limiting the analysis to the selection. The selection is delineated by a boundary drawn around it. The support conditions along this boundary are user defined. In most cases, simple support, or no support is applicable. For better results, the boundary is drawn somewhat larger than the slab region of interest, in order to minimize the impact of the user imposed conditions on the vibrations of the interest region. The following example illustrates the point.

Consider the vibration of the panel identified in Figure EX1. The objective is to determine whether the likely vibration of the identified panel is acceptable.

For illustration, the traditional method of considering the entire floor system is used first. This is followed by isolating the region of investigation, using the Floor Pro's "Excluder" feature.

Analysis Using the Entire Floor

The entire 3D floor model is discretized (Fig. EX4) and analyzed to extract its first few frequencies and modes of vibration. Its first three modes of vibrations and frequencies are illustrated in Fig. EX5. It is noted that in all three cases, the primary excitation of the floor is in regions other than the panel of primary interest. Thus casting doubt whether any of the three could be considered as the primary mode of vibration of the interest region.

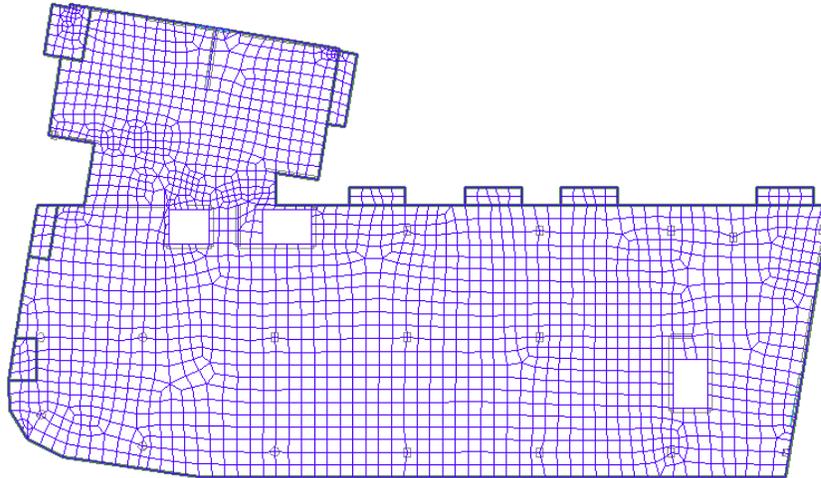
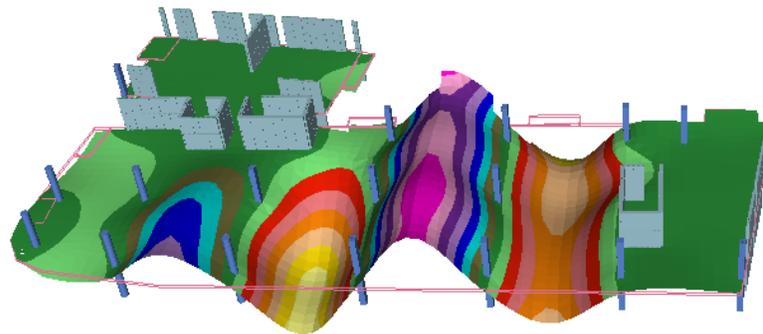
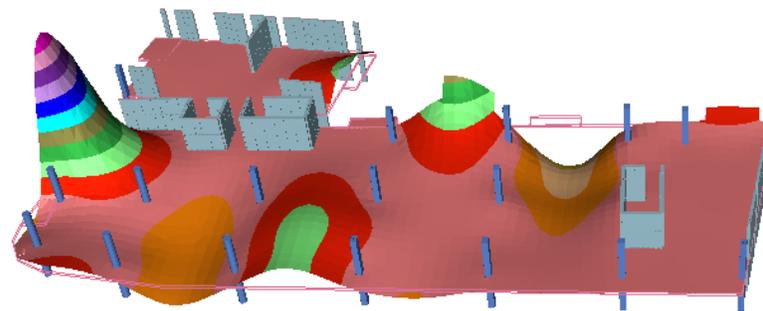


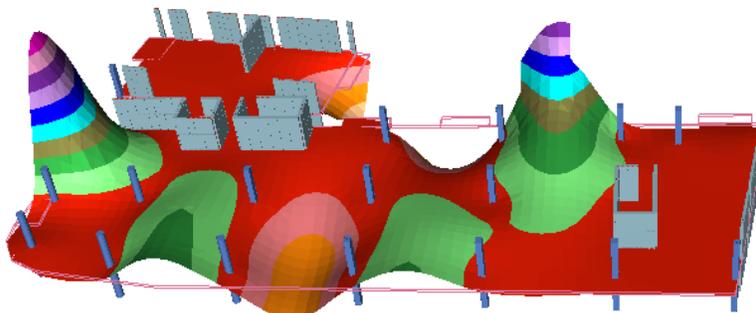
FIGURE EX4 – DISCRETIZATION OF THE ENTIRE FLOOR MODEL



(a) First mode – frequency 5.97 Hz



(b) Second mode – frequency 6.33 Hz

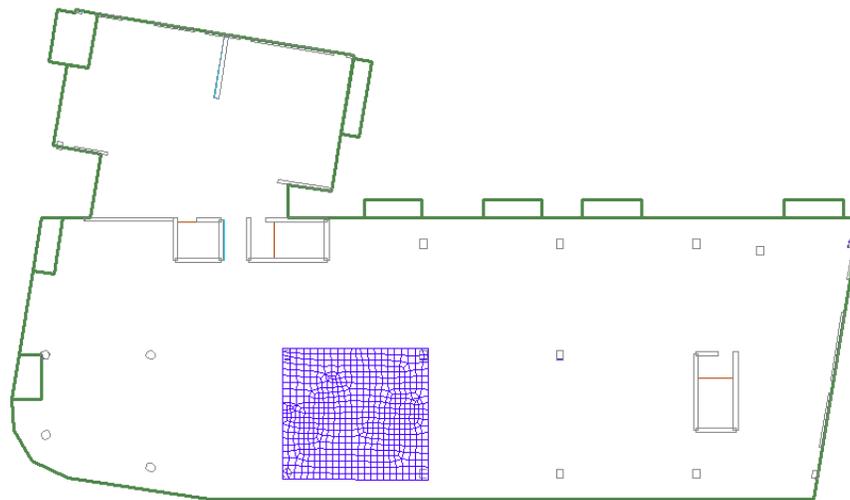


(c) Third mode – frequency 6.44 Hz

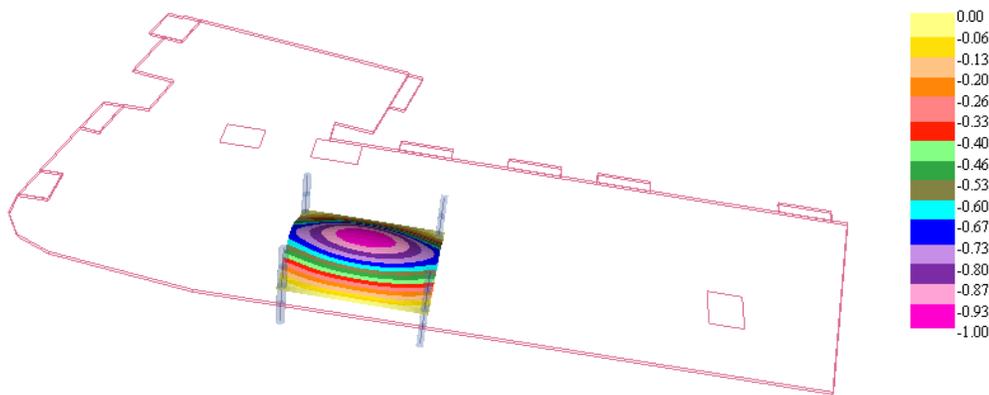
FIGURE EX5 THE FRIST THREE MODES AND FREQUENCIES

Analysis Using the Region of Interest Only

In this case the region selected is limited to the panel of interest. The panel is isolated from the rest of the floor system by the boundary lines drawn around it. The support conditions assumed for the boundary can have a significant impact on the frequency of the sub-region. In Fig. EX6, the boundaries are assumed to be simple supports (zero translational displacement, free to rotate). This boundary condition can be too restrictive and results in a higher frequency. To obtain a more reliable solution, the boundary of the selected region should be extended farther away from the panel of interest, in order to minimize the impact of the user selected boundary restraints. This is illustrated in the next treatment.



(a) Identification of sub-region of interest

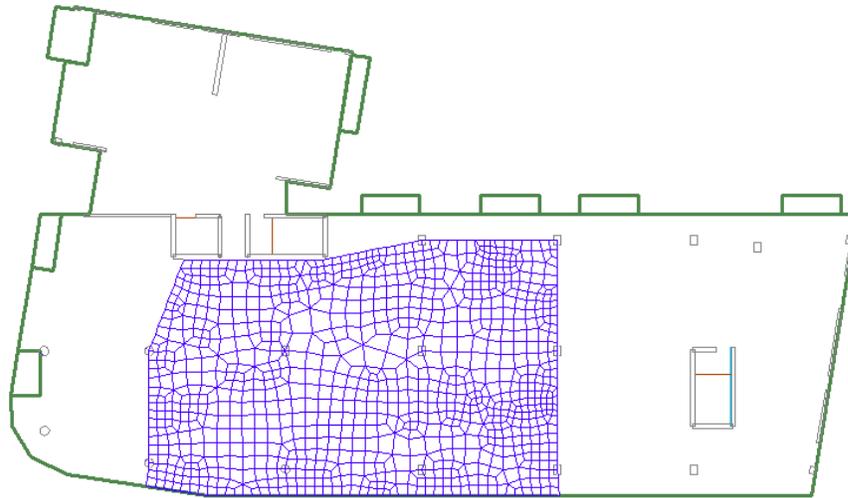


(b) First mode of vibration – frequency 7.95 Hz

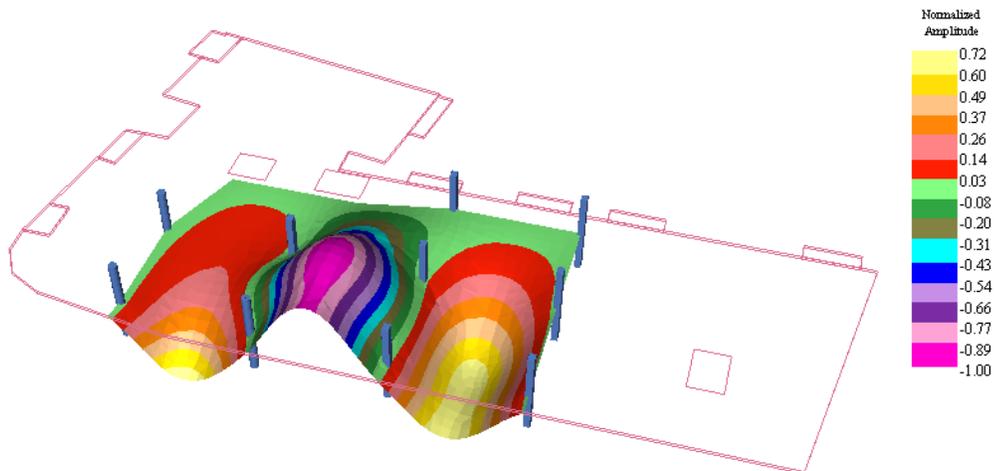
FIGURE EX6 IDENTIFICATION OF SUB-REGION AND ITS FIRST MODE OF VIBRAITON

Analysis Using the Extended Region of Interest

The extended region selected (Fig. EX7-a) stretches one span on each side beyond the panel of interest. The associated frequency and mode shape are reported in part (b) of the figure. The frequency obtained in this modeling is deemed to be more reliable to form the basis for acceptability of probable vibrations to be experienced by the panel of interest.



(a) Selection of an extended region



(b) First mode of vibration – Frequency 6.07 Hz

FIGURE EX7 SELECTION AND VIBRATION OF AN EXTENDED REGION

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