STRAIN COMPATIBILITY FOR DESIGN OF PRESTRESSED SECTIONS

Concrete sections are not always rectangular or simple T, with one layer of prestressing tendons and another layer of rebar. In the general case, in particular in floor systems, the geometry of a section selected for design maybe non-standard. Further, there are generally multiple tendons crossing a section at different locations and at different angles. Similarly the nonprestressed steel is likely to be multi-layered and not normal to the section. This is particularly true, where the design engineer selects a mesh of reinforcement throughout the floor system, and bases the design on the reinforcement needed to supplement the area of the mesh selected.

This Technical Note illustrate the concept of strain compatibility, as applied to design of prestressed sections in bending.

Figure 1 shows the geometry, strain and force distribution for a rectangular section reinforced with prestressing and nonprestressed steel. At strength limit state, the forces developed in the various components of the section will add up to provide the nominal capacity of the section M_n. The nominal capacity is determined from the forces shown in part (b) of the figure using the following common relationships:

$$C = T \qquad\qquad\qquad (1)$$

$$M_n = \text{sum of moment of the internal forces} \qquad (2)$$

Where,

C = Total compression on the section; and
T = total tension on the section.

The force in each component of the section is based on its strain and stress-strain relationship. Figure 2 shows the stress-strain relationships commonly assumed.

The distribution of strain over the section is assumed linear. At strength limit state, the section's capacity is determined based on an assumed maximum strain at the compression fiber of concrete. ACI-318 sets the maximum concrete strain at 0.003. Other building codes assume the same or larger values. For example, the British code (BS 8110) assumes 0.0035.

With the value of maximum compression strain in concrete assumed as constant, and the distribution of strain over the section linear, the depth of the compression zone (c) that satisfies Eqn (1) is determined. The determination of "c" involves an iterative procedure.

Once the value of c is determined, the strain in each of the components of the section is simply given by the linear distribution of strain shown for the "limit state" diagram of Fig. 1, except for prestressing tendons. For prestressing tendons the following applies:

$$\varepsilon_{ps} = \varepsilon_{se} + \varepsilon_{ce} + \varepsilon_{p} \qquad\qquad (3)$$

Where,

ε_{ps} =   Strain in prestressing steel at strength limit state, that is when section develops its nominal moment;

ε_{se} =   strain in prestressing steel due to effective stress in tendon after all losses have taken place (zero dead and live load assumption);

ε_{p} =   strain in concrete at level of prestressing steel centroid when section develops its nominal strength; and

ε_{ce} =   concrete strain at level of tendon centroid, after all tendon losses have taken place and with assumed dead and live loading. This strain is also referred to as decompression strain.

Some codes, such as the European code (EC2) and many engineers allow the value of ε_{ce} to be assumed as zero for simple calculations.

Once the nominal moment is calculated, the design moment is obtained using the following relationship.

$$M_{u} = \Phi M_{n} \qquad\qquad (4)$$

Where, a tendon or a nonprestressed steel makes an angle with the section being designed, as is often the case in profiled tendons and floor slabs, the calculated strain for the location of the tendon or rebar is modified as follows:

$$\varepsilon_{effective} = \varepsilon_{normal} * \cos^{2}\theta$$

Where,

$\varepsilon_{effective}$ =   Strain used to determine the force in the component for the calculation of nominal strength;

ε_{normal} =   strain in the component calculated from the linear distribution of strain over the section; and

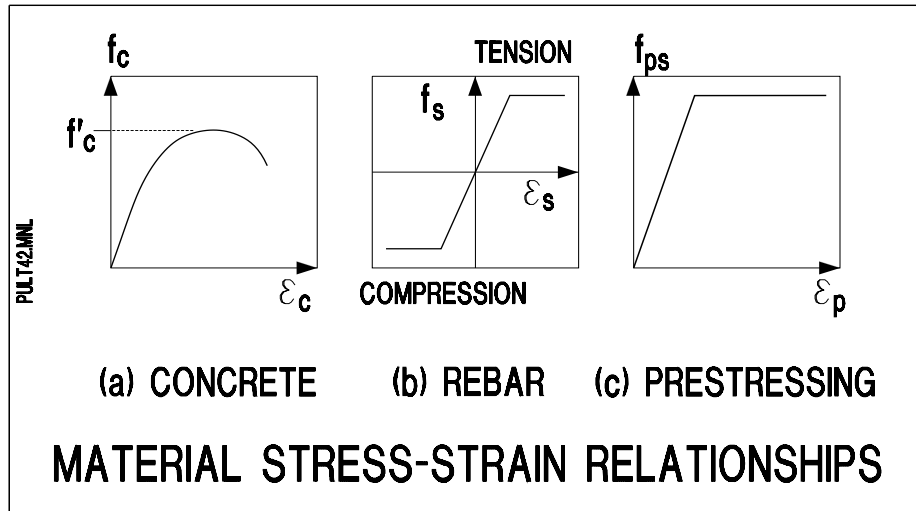θ =   angle between the component and normal to the section being designed.
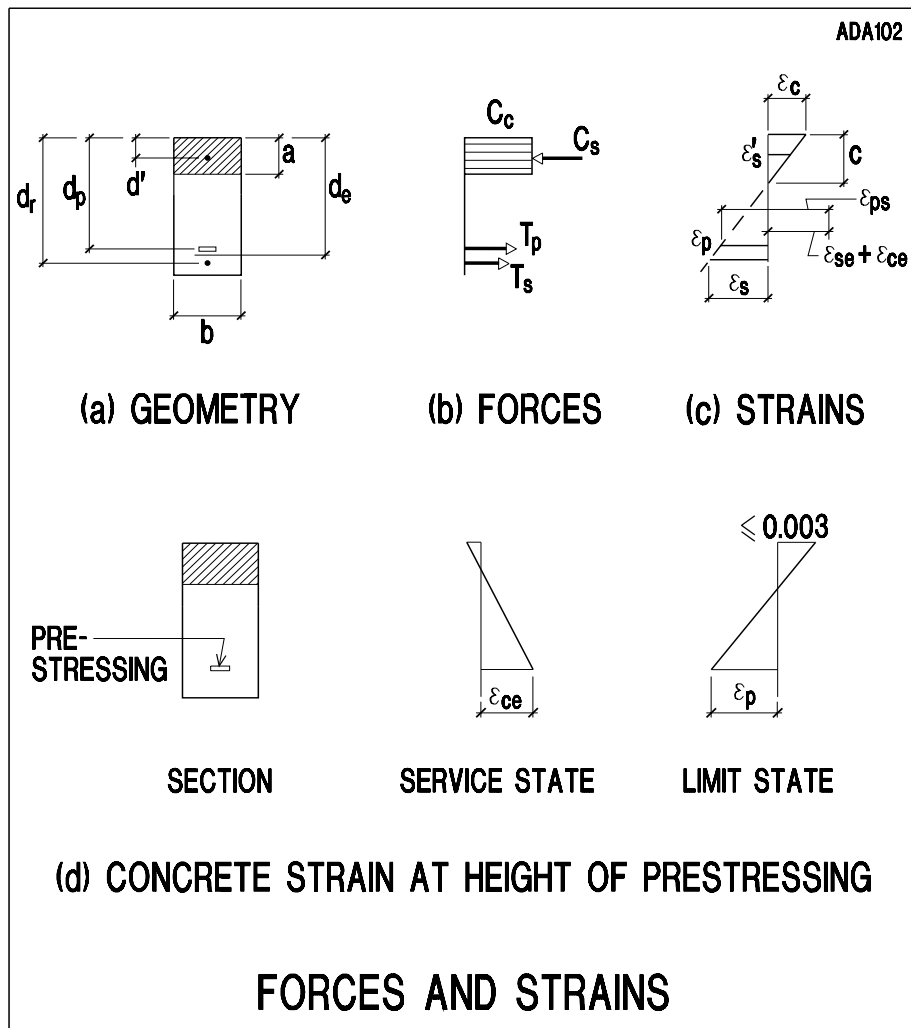
FIGURE. 1

FIGURE. 2

NUMERICAL EXAMPLE

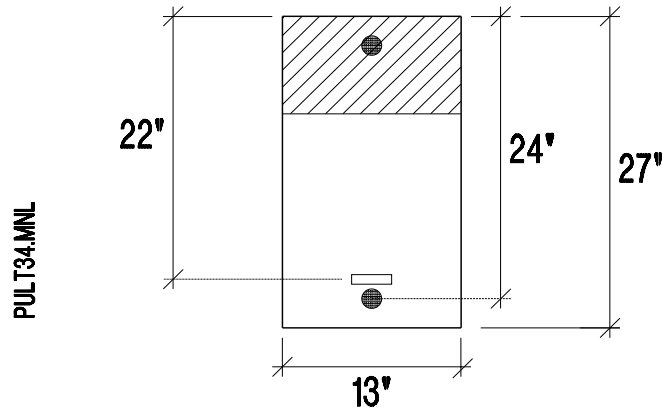Using strain compatibility, determine the design capacity (ΦM_n) of the section shown below:

FIGURE EX-1   GEOMETRY OF SECTION

Given:

Geometry:
depth (h)	= 27 in
stem width (b)	= 13 in
d_p	= 22 in
$d_r = d_t$	= 24 in

Concrete:
f'_c	= 4000 psi
E_c	= 3605 ksi

Prestressing:
3 – ½ in strand; Aps	= 3*0.153 = 0.459
Grouted system	
fpu	= 270 ksi
E_{ps}	= 28000 ksi
f_{se}	= 175 ksi (effective stress after all losses)

Rebar:
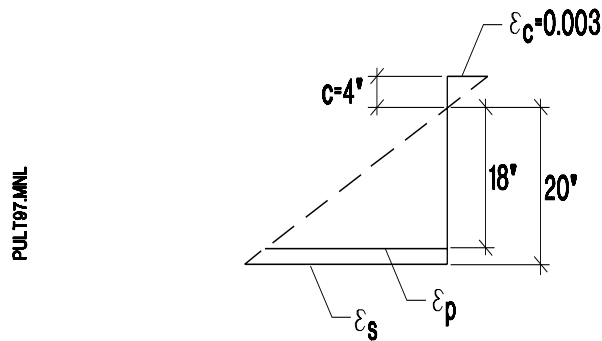|	
Fy	= 60 ksi
Es	= 29000 ksi
1 # 6; As	= 0.44 in^2

Required:

ΦM_n

FIGURE EX-2 DISTRIBUTION OF STRAIN AT STRENGTH LIMIT STATE

1 – Determine the depth of neutral axis
>   Using trial and error, the depth of neutral axis "c" that satisfies the equilibrium of forces (T=C) is determine to be 4 in. The distribution of strain over the section, based on maximum concrete strain (0.003) and 4 in depth of the neutral axis is shown in **Fig. EX-2**.

2 – Determine the strains

2.1 Rebar

ε_s =   (20/4) * 0.003   0.015 > (f_y/E_s) = 60/29000 = 0.00207

Hence, rebar yields

2.2 Prestressing

ε_{ps} = ε_{se} + ε_{ce} + ε_p

ε_{se} = f_{se}/E_{ps} = 175/28000 = 0.00625

ε_{ce} = F_{pt} (1/A + e^2/I) / E_c

>   Where,

>   A = Gross area of cross section;
>   e = eccentricity of tendon with respect to the centroid of gross cross section; and
>   I   = moment of inertia of section.

>   A = 13 * 27 = 351 in^2

>   I = 13 * 27^3 /12 = 21323 in^4

F_{pt} = 175 * 0.459 = 80.33 k

$e = 22 - 27/2 = 8.5$ in

$\varepsilon_{ce} = 80.33 [(1/351) + (8.5^2/21323)] / 28000 = 1.78$ E-5

$\varepsilon_p = (18/4) * 0.003 = 0.0135$

$\varepsilon_{ps} = 0.00625 + 0.0135 + 0.0000229 = 0.0198$

$f_{ps} = \varepsilon_{ps} * E_{ps} = 0.0198 * 28000 > 270$ ksi ; use 270 ksi

3 – Determine the forces

$a = 0.85 * 4 = 3.4$ in

$C_c = 0.85 * 4 * 13 * 3.4 = 150.28$ k

$T_s = 0.44 * 60 = 26.4$ k

$T_{ps} = 3 * 0.153 * 270 = 123.93$ k

4 – Check equilibrium

$C = C_c = 150.28$ k

$T = T_s + T_{ps} = 26.4 + 123.93 = 150.33$ k

$C = T$ OK

5 – Calculate design capacity

$c/d_t = 4/24 = 0.167 < 0.375$, hence $\Phi = 0.9$

Take moments about the centroid of compression block

$\Phi M_n = 0.9 * [123.93 (22 - 3.4/2) + 26.35 (24 - 3.4/2)] = 2793$ k–in

$\Phi M_n = 2793/12 = 232.75$ k-ft