ADAPT Technical Note

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Effective Width in Post-Tensioned Flanged Beams

This explains the effective width to be used in the stress calculation of post-tensioned T-beams. Simple beam formulas are used in the practical design of flange beams. The effective width concept was introduced to make the calculated maximum stress from the simple beam formula to be the same as that obtained from an elastic solution in which the plate action of the flange is recognized.

Figure 1a shows a slab supported on widely spaced parallel beams. The loading is uniform. The compressive stresses in the flange are highest at the flange/stem intersection and taper off away from the beam stem. This is due to the shear lag phenomenon. For simplification in the calculations, the nonuniform compression over the tributary of the flange can be replaced with a rectangular block of equal force, having a width smaller than the tributary, and a stress equal to the maximum. The equal force rectangular block defines the “effective width” in bending. For uniform loading on simply supported beams, the effective width is eight times the slab thickness on each side of the stem. Hence, the maximum bending stress can be calculated from the simple beam formula as follows:

$$f = M_b * c / I_b \qquad (1)$$

Where, I_b is the moment of inertia of the portion of the beam bounded by the effective width (hatched area in Fig. 2b). The moment of inertia is calculated about the centroid (point B) of the effective width.

In post-tensioned beams, the tendon force is deposited at the end of the beam (slab edge). The tendon force creates an axial force and a moment, if the force is eccentric to the centroid of the tributary area. An axial force at the slab edge spreads sharply over the entire cross-sectional area of the slab (Fig. 1c). The force diffuses into a uniform compression away from

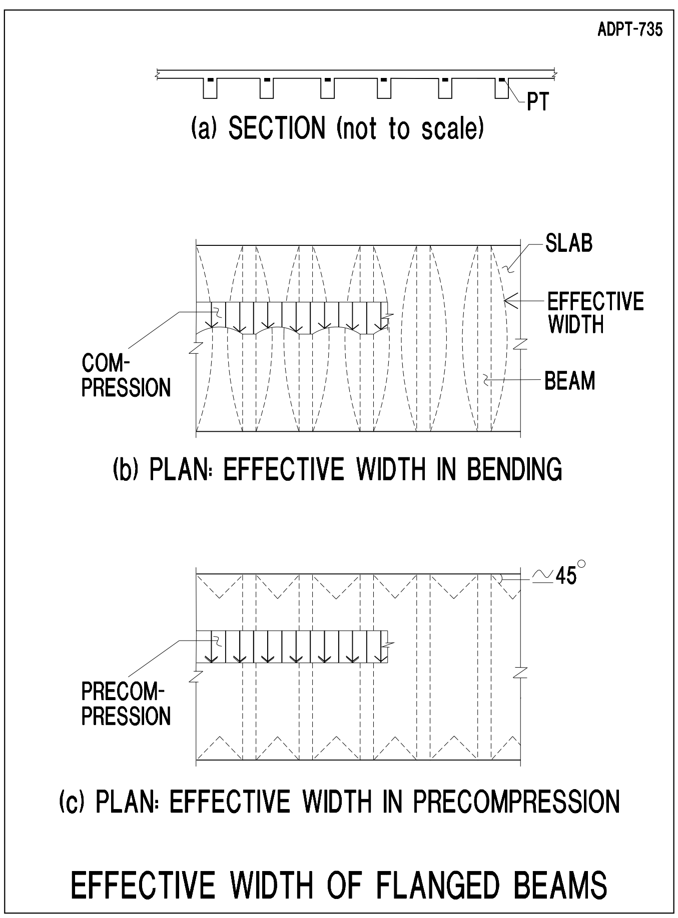

FIGURE 1

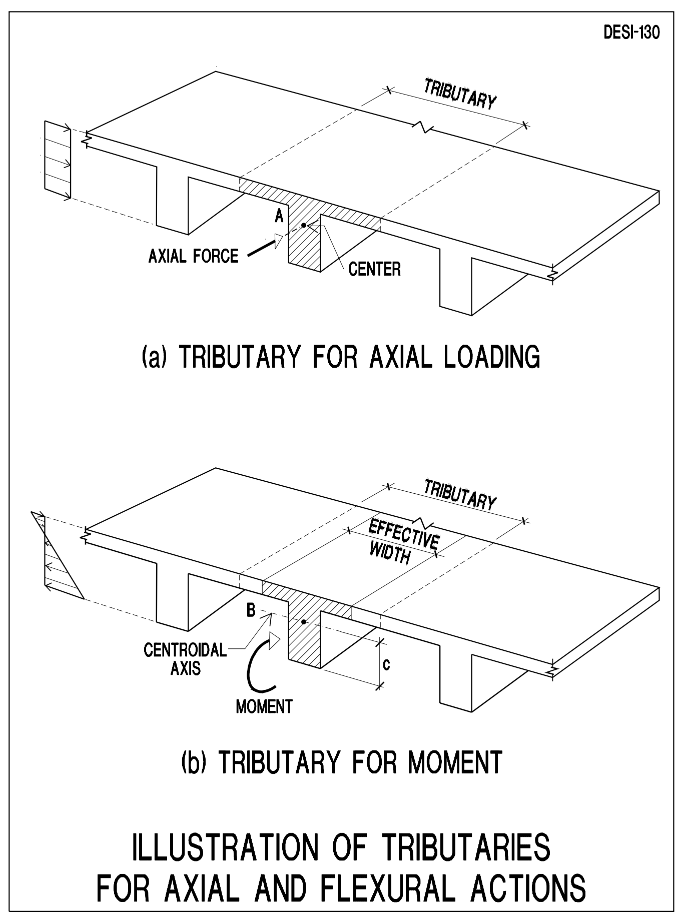

FIGURE 2

the of anchorage devices. The distribution into a uniform stress over the entire tributary is based on the fact that a load applied at the centroid of a section divides uniformly over the entire cross section away from the point of application of the load. Hence, at any given section along the length of the member, the stress due to prestressing is:

$$f_{pt} = M_{pt} * c/I_b + P/A \qquad (2)$$

Where, M_{pt} is the moment due to post-tensioning. For a simply supported beam, M_{pt} is equal to the post-tensioning force times its eccentricity from the centroid of the entire tributary (point A in Fig. 2a). Like before, I_b and c are moment of inertia and distance from the centroid of the reduced area as shown in Fig. 2b. A, is the area of the entire tributary (hatched in Fig. 2a).

The combined stress at a point distance “c” from the centroid of the effective width area is given by:

$$f = f_b + f_{pt} = (M_b + M_{pt})*c/I_b + P/A \qquad (3)$$

EXAMPLE

Figure EX-1 shows the partial plan of a beam and slab construction. The beams are single span and simply supported. Each beam is stressed with 16 – ½ in (13 mm). strand providing a combined force of 428.2 k (1904.72 kN) at the beam’s mid-span, where the centroid of the tendon is 2.75 in. (70 mm) from the soffit. The sum of dead and live moments at midspan is 1013.83 k-ft (1374.55 kNm).

It is required to calculate the midspan stress at the beam soffit. Refer to Fig. EX-2.

Area the beam tributary:

A = 204x5 + 14x25 = 1370 in^2 (883869 mm^2)

Distance of centroid from soffit: = 23.67 in. (601 mm)

Eccentricity of tendon with respect to centroid (point A) of the tributary = 20.92 “ (531 mm)

Moment from post-tensioning:

M_{pt}= 428.2x20.92/12 = 746.50 k-ft (1012.10 kNm)

Average precompression:

P/A = 428.2x1000/1370 = 312.55 psi (2.15 MPa)

Area of beam reduced by effective width:

A = 94x5 + 14x25 = 820 in^2 (529031 mm^2)

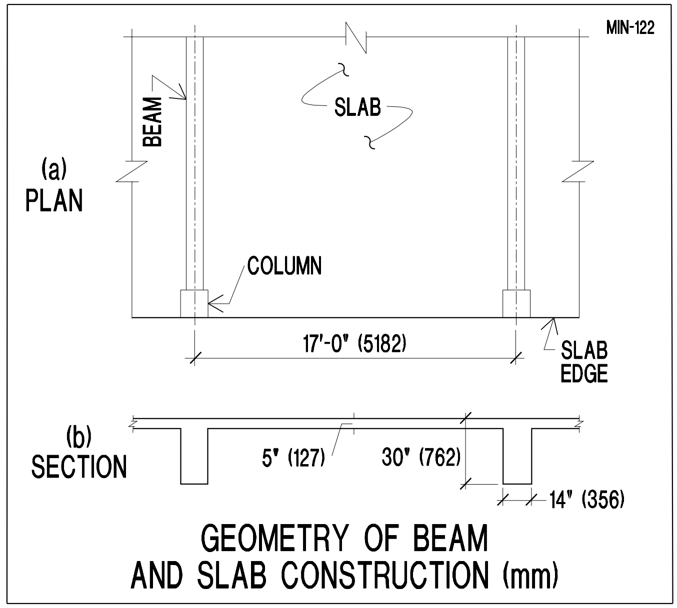

FIGURE EX-1

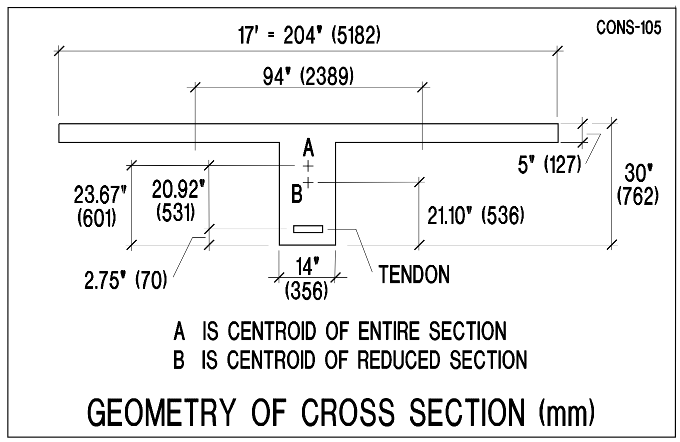

FIGURE EX-2

Distance of centroid (point B) from soffit: = 21.10 in. (535.94 mm)

Moment of inertia of section about axis through B

I_b = 64345.53 in^4 (2.6783x10^{10} mm^4)

Stress at soffit: $f = f_b + f_{pt} = (M_b + M_{pt})*c/I_b + P/A$

f = (1013.83 - 746.50)x12x1000x 21.10/ 64345.53 – 312.55 = 739.39 psi (5.10 MPa)

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